THE NUMERICAL SOLUTION AND ANALYSIS OF AIRPLANE SPIN EQUATIONS MODELED IN A FIXED COORDINATE SYSTEM

Robert Louis Champoux

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

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of Airplane Spin Equations
Modeled in a Fixed Coordinate System

by

Robert Louis Champoux

Thesis Advisor: Louis V. Schmidt

December 1972

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Robert Louis Champoux
Lieutenant, United States Navy
B.S., United States Naval Academy, 1965

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
December 1972

ABSTRACT

Three forms of the airplane spin equations of motion, derived by Buehler in Reference [1], form the basis for the development of a computer program designed to seek dynamically stable equilibrium solutions of a spinning aircraft. The program incorporates two solution techniques: one based upon Euler integration, the other, a version of minimization by gradient search. Secondary programs are developed to (1) generate power-off glide parameters for use in the validation of the equations of motion, and (2) evaluate equation residuals obtained from a grid of initial conditions over the potential solution space. F-111 and F-4 aerodynamic force and moment models were utilized to evaluate the solution methods and equations of motion. The numerical results indicate that the F-111 and F-4 data are not representative of the actual aircraft and, therefore, it is highly unlikely that dynamically stable equilibrium solutions can be achieved from these models. The utility of the two solution methods is evaluated and the numerical results are analyzed in order to gain insight into the optimal application of the three forms of the equations of motion. The paper concludes with a discussion concerning the qualitative validation of the equations of motion.

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TABLE OF SYMBOLS

The definitions of the symbols used in this paper are as follows:

C_g	Center of gravity
$\bar{c}$	Mean aerodynamic chord, feet
g	Acceleration of gravity, ft/sec^2
I_x, I_y, I_z	Airplane principal moments of inertia, slug-ft^2
M	Airplane mass, slugs
N	Load factor; number of "g's"
R	Spin Radius, feet
S	Criterion function value; equal to the sum of the absolute values of the equation residuals, SUM
s	Airplane wing area, ft^2
t	Time, seconds
Δt	Increment of time, seconds
V_{cg}	Airplane cg velocity, ft/sec
X, Y, Z	Airplane principal axis system
X_1, Y_1, Z_1	Reference Cartesian axis system
Z_o	Airplane altitude, feet
$\dot{Z}_o$	Rate of ascent, ft/sec
α	Airplane angle of attack, alpha
α_{ij}	Direction cosines in terms of Euler angles (Table I)
β	Airplane sideslip angle, Beta
$\dot{\gamma}$	Spin rate, per second
$\delta_a, \delta_r, \delta_e$	Aileron, rudder and elevator deflections, degrees
Θ, Ψ, Φ	Airplane principal axis system orientation Euler angles with respect to the X_1, Y_1, Z_1 system

Θ_p, Ψ_r, Φ_y Airplane orientation in pitch, roll and yaw with respect to the X_1, Y_1, Z_1 system

ρ Air density, slugs/ft^3

W Aircraft weight, lbs

FOREWORD AND ACKNOWLEDGMENTS

The purpose of this effort was to develop a computer program which would numerically predict the dynamically stable spin characteristics of an aircraft. Essential to the task was the analysis and validation of the airplane spin equations derived by LCDR R. E. Buehler and described in Reference [1]. Although the final results were inconclusive, if any success is ultimately realized from the program it will be due in a large part to LCDR Buehler who, during the concurrent development, withstood many frontal assaults upon the veracity of his equations. He many times patiently rederived various aspects of his work only to learn that the intricacies of the computer program were the underlying culprits. Appreciation is also expressed to Professor L. V. Schmidt, who significantly aided in the validation of the equations and for providing constructive criticisms concerning manuscript preparation. The author is also indebted to Professors Gawain and Redlin of the NPS faculty whose council helped in the utilization of the gradient method and criterion search aspects of the analysis. Finally, the staff of the William Randolph Church Computer Center is to be commended for the consultative support which they provided.

I. INTRODUCTION

A. BACKGROUND

Aircraft flight tests frequently reveal significant differences between the predicted and the actual spin characteristics of high performance systems. Such differences often result in expensive engineering changes to the aircraft design and tend to reduce the mission capability of the aircraft. Furthermore, if the final spin characteristics are such that pilots are not permitted to practice the recovery from intentional spins, then many will consider the outer regions of the performance envelope with trepidation resulting in further degradation of the weapon system's effectiveness. The need therefore, to accurately predict the spin characteristics of proposed aircraft designs is of paramount importance.

Historically, efforts towards spin performance prediction have been concentrated in two areas: the development of system time histories and the analysis of free-spinning model test results. System time histories involve the simultaneous solution of the aircraft equations of motion with the required force and moment coefficients being determined from wind tunnel model data. Solutions are obtained using digital computer programs which numerically integrate the appropriate equations of motion. There has been minimal success towards achieving a time history method that will accurately predict spin performance due to the critical dependence upon entry conditions. Free-spinning model tests utilize instrumented dynamically scaled models which are generally deployed into vertical wind tunnels. This method however has a limited capability in simulating the full scale

spin environment (Reynolds, Mach, and Froude Numbers and the spin entry mechanism) and often the tests fail to identify all of the possible spin modes that may be experienced by the full scale vehicle.

B. GENERAL

This paper represents the final report on the development of a digital computer program designed to analytically predict the airplane spin performance using a unique set of aircraft equations of motion and simplifying assumptions regarding the nature of spin mode solutions. The primary hypothesis upon which the development of the computer program was predicated was the unique dynamically stable equilibrium (steady-state) solution for each aircraft spin mode. This assumption implies that all motion will ultimately be damped to a steady-state solution. Using the above hypothesis, the dynamically stable equilibrium solution may be determined directly without the encumbrances of a time history approach, with its required dependence upon entry conditions. The hypothesis ignores, however, the possible (and perhaps most probable) existence of a dynamically unstable equilibrium solution which could also yield a stable trajectory in 'state space'.

C. SCOPE

A rigorous research effort into the development of a means of predicting airplane spin modes would involve the following stages:

I. Literature search

II. Derivation of the mathematical model

III. Verification of the mathematical model by using time history comparisons with baseline data

IV. Development of a computer program designed to seek spin solutions

V. Verification of the computer program by utilizing wind tunnel data modified for simulator use

VI. Determine and/or predict "real world" spin modes.

Stages I and II are described in References [7] and [1]; while the base line data required for phase III is included in Reference [2]. The purpose of this effort was an attempt to accomplish steps IV and VI. This rather large undertaking was conducted concurrently with LCDR Buehler, who developed the applicable equations and coordinate systems required by stage II. The limited time available precluded a numerical validation of the computer program (phase III).

II. ANALYTIC CONSIDERATIONS

A. COORDINATE SYSTEM

The coordinate system used in the mathematical model is a combination of the airplane principal axis system and a fixed cylindrical system. The vertical (z) axis of the fixed cylindrical system represents the initial central axis of motion. The cylindrical system locates the aircraft center of gravity in terms of the altitude coordinate (z_o), the spin radius coordinate (R) and the angular position coordinate (γ). The orientation of the aircraft with respect to the cylindrical system is described in terms of Euler angles. A cartesian coordinate system is fixed at the cg position on the R vector with its axis (x_1) in the (-R) direction and the (z_1) axis is oriented in the (-z) direction. The cartesian system provides a reference for the orientation Euler angles whereby zero values of (Θ), (Ψ), and (ϕ) would yield an upright, wings level airplane with its principal (x) axis pointed inward in the (-R) direction. Figure 1 depicts the details of the coordinate system; the positive directions of the position coordinates are also shown. As Buehler noted "The rationale behind this choice of coordinate system is that it more simply (in a mathematical sense) represents the motion being modeled." [Reference 1]. The advantage of this hybrid coordinate system is that it obviates a steady-state spin solution. The motion of the spinning aircraft is easily recognized by constant values of the six independent coordinate variables (R, $\dot{z}_o$, $\dot{\gamma}$, Θ, Ψ, ϕ). Additionally the complexities of the stall, departure, and incipient spin motion have no impact on the results since the objective is to determine the steady-state spin modes.

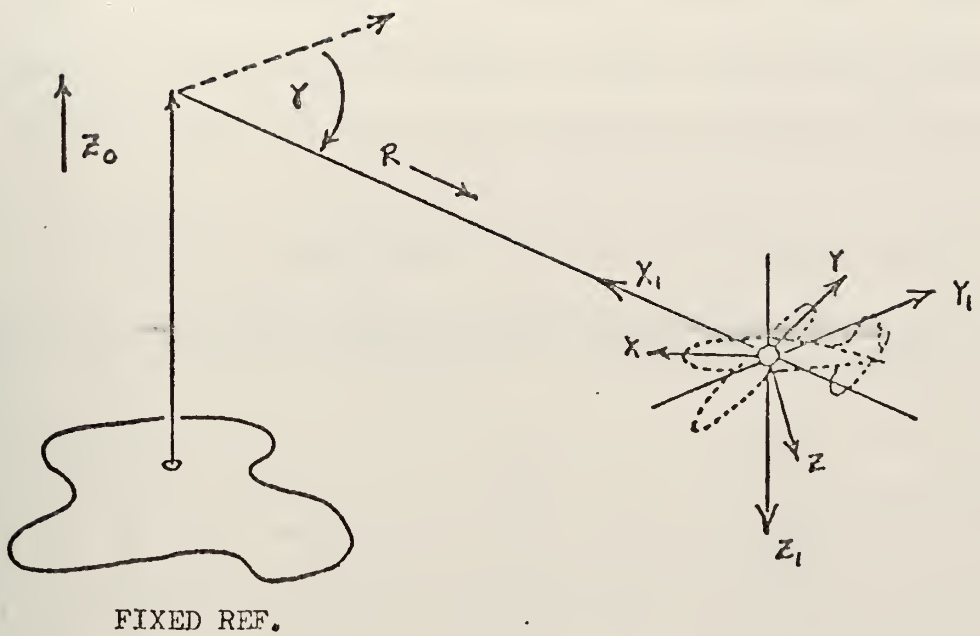

Fig. 1. The Coordinate System Used to Model the Spin

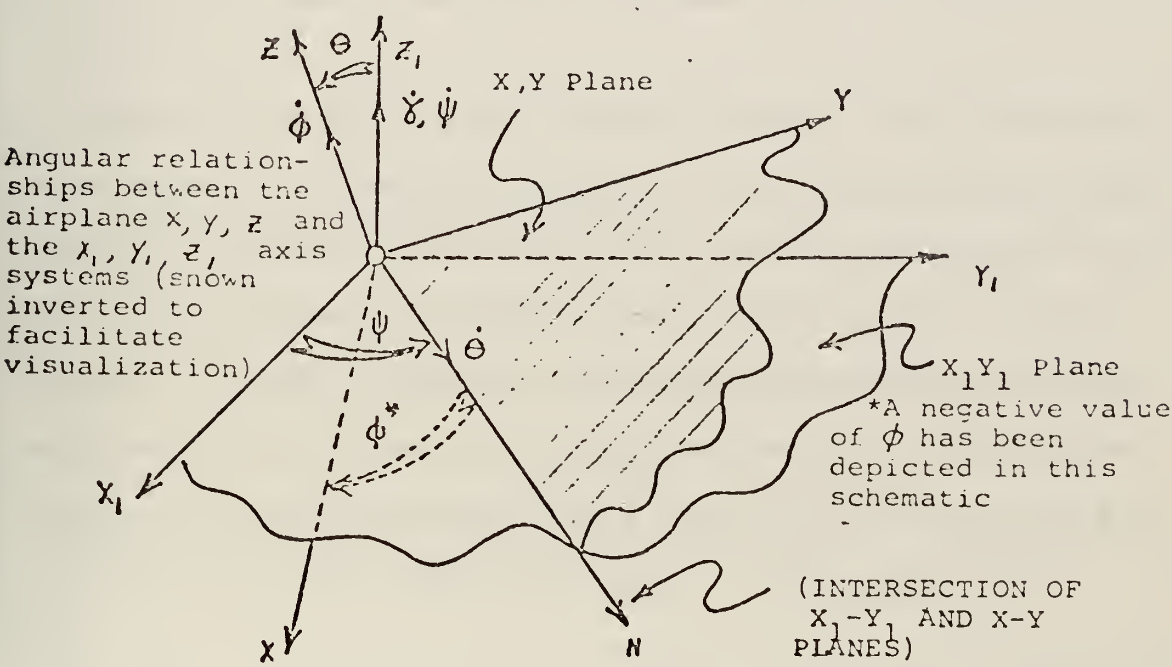

Fig. 2. A General Schematic of Euler Angle Relationships

B. EULER ORIENTATION ANGLES

The Euler Angle relationships utilized in the mathematical model involve rotations which are measured relative to the vertical (z_1) axis and to the intersection of the (x, y) plane and the (x_1, y_1) plane. This system yields equations for the direction cosines as listed in Table I.

Table I: Direction Cosines in Terms of Euler Angles

Cosines of Angles between X, Y, Z and X_1, Y_1, Z_1			
	X	Y	Z
X_1	$\alpha_{11} = \cos\phi\cos\psi - \sin\phi\sin\psi\cos\theta$	$\alpha_{21} = -\sin\phi\cos\psi - \cos\phi\sin\psi\cos\theta$	$\alpha_{31} = \sin\theta\sin\psi$
Y_1	$\alpha_{12} = \cos\phi\sin\psi + \sin\phi\cos\psi\cos\theta$	$\alpha_{22} = -\sin\phi\sin\psi + \cos\phi\cos\psi\cos\theta$	$\alpha_{32} = -\sin\theta\cos\psi$
Z_1	$\alpha_{13} = \sin\theta\sin\phi$	$\alpha_{23} = \sin\theta\cos\phi$	$\alpha_{33} = \cos\theta$

Inherent in this orientation system, however, is the difficulty in visualizing the final orientation without employing sketches or digital graphics equipment. This can be readily appreciated by referring to the general schematic of the Euler angle relationships depicted in Figure 2. In order to alleviate the visualization problem, Buehler, in Reference [1], derived relationships between a set of orientation angles with a sequence of pitch (Θ_p), roll (Ψ_r), and then yaw (Φ_y) and the Euler angles depicted in Figure 2.[1] The equations for

[1] Hereafter, and in the computer program, the orientation angles will be referred to as the 'ordered' Euler angles, while the orientation angles depicted in Figure 2 will be referred to as the 'reference' Euler angles.

converting the ordered Euler angles to the reference Euler angles are:

$$\phi = \psi_y - \tan^{-1}\left\{\tan\left(\frac{\theta_p}{2}\right)\tan\left(\frac{\phi_R}{2}\right)\right\} - \tan^{-1}\left\{\frac{\tan\left(\frac{\theta_p}{2}\right)}{\tan\left(\frac{\phi_R}{2}\right)}\right\} \tag{1}$$

$$\psi = \tan^{-1}\left\{\frac{\tan\frac{\theta_p}{2}}{\tan\frac{\phi_r}{2}}\right\} - \tan^{-1}\left\{\tan\left(\frac{\theta_p}{2}\right)\tan\left(\frac{\phi_R}{2}\right)\right\} \tag{2}$$

$$\theta = 2\tan^{-1}\left\{\frac{\sin\left[\tan^{-1}\left\{\tan\left(\frac{\theta_p}{2}\right)\tan\left(\frac{\phi_r}{2}\right)\right\}\right]}{\sin\left[\tan^{-1}\left\{\frac{\tan\left(\frac{\theta_p}{2}\right)}{\tan\left(\frac{\psi_r}{2}\right)}\right\}\right]\tan\left(\frac{\phi_r}{2}\right)}\right\} \tag{3}$$

The equations for obtaining (θ_p), (ψ_r) and (ϕ_y) from the reference Euler angles are:

$$\psi_y = \phi + \tan^{-1}\left\{\frac{\tan\left(\frac{\psi}{2}\right)\sin\left(\frac{\pi/2-\theta}{2}\right)}{\sin\left(\frac{\pi/2+\theta}{2}\right)}\right\} + \tan^{-1}\left\{\frac{\tan\left(\frac{\psi}{2}\right)\sin\left(\frac{\pi/2-\theta}{2}\right)}{\cos\frac{\pi/2+\theta}{2}}\right\} \tag{4}$$

$$\theta_p = \tan^{-1}\left\{\frac{\tan\left(\frac{\psi}{2}\right)\cos\left(\frac{\pi/2-\theta}{2}\right)}{\cos\left(\frac{\pi/2+\theta}{2}\right)}\right\} - \tan^{-1}\left\{\frac{\tan\left(\frac{\psi}{2}\right)\sin\left(\frac{\pi/2-\theta}{2}\right)}{\cos\left(\frac{\pi/2+\theta}{2}\right)}\right\} \tag{5}$$

$$\Phi_R = \frac{\pi}{2} - 2\,\mathrm{TAN}^{-1}\, \frac{\mathrm{SIN}\left[\mathrm{TAN}^{-1}\left\{\dfrac{\mathrm{TAN}\left(\frac{\psi}{2}\right)\mathrm{SIN}\left(\frac{\pi/2-\theta}{2}\right)}{\mathrm{SIN}\left(\frac{\pi/2+\theta}{2}\right)}\right\}\right]}{\mathrm{SIN}\left[\mathrm{TAN}^{-1}\left\{\dfrac{\mathrm{TAN}\left(\frac{\psi}{2}\right)\mathrm{COS}\left(\frac{\pi/2-\theta}{2}\right)}{\mathrm{COS}\left(\frac{\pi/2+\theta}{2}\right)}\right\}\right]\mathrm{TAN}\left(\frac{\pi/2-\theta}{2}\right)} \tag{6}$$

The above equations have been numerically verified and validated for quadrants I and IV ($\pm 90^{\circ}$). This is sufficient for an aircraft oriented in an upright spin, however, for the case of an inverted spin where the aircraft will encounter quadrants II and III, the above relationships are invalid.

C. THE EQUATIONS OF MOTION

The spin is an uncontrolled large angle, six degree of freedom motion experienced by an aircraft in the stalled aerodynamic region. Since the spinning phenomenon is a complex motion which is influenced by a host of nonlinear variables, the equations which describe that motion are both nonlinear and coupled. Buehler, in Reference [1], derived the aircraft spin equations of motion utilized in the computer program. The equations and modifications thereto are described below.

1. The Full Equations

The equations of motion were cast in a more useable form by solving each equation for the corresponding acceleration parameter. This form avoids obscuration of the physical nature of the problem and places the numerical analyst in a better position to monitor, trouble-shoot and interpret iterative solution results.

2. The Modified Equations

The full equations were further modified by recognizing that only dynamically stable (or quasi-stable) equilibrium solutions were sought. Thus angular orientation rates ($\dot{\Theta}, \dot{\Psi}, \dot{\Phi}$) were equated to zero, which in turn eliminated a considerable number of terms. The R parameter was retained because it effects the values of α, β, and V_r to the extent that when ignored, it seriously impeded the iterative solution effort.

3. The Short Equations

The modified equations were further simplified by equating the R parameter to zero. This form is useful when seeking purely steady state solutions using the more mathematically elegant solution methods such as the gradient search method described in Section H.

The full, modified, and short equations are listed in Appendix I.

D. ANCILLIARY RELATIONSHIPS AND DEPENDENT VARIABLES

In order to utilize conventional aerodynamic force and moment coefficients in the equations of motion, certain ancilliary relationships were needed. As derived by Buehler, and as depicted in Figure 3:

Angle of Attack:

$$\alpha = TAN^{-1}\left\{\frac{\alpha_{31}\dot{R} + \alpha_{32}\dot{\gamma}R + \alpha_{33}\dot{Z}_o}{\alpha_{11}\dot{R} + \alpha_{12}\dot{\gamma}R + \alpha_{13}\dot{Z}_o}\right\} \tag{7}$$

Sideslip Angle:

$$\beta = TAN^{-1}\left\{\frac{\alpha_{21}\dot{R} + \alpha_{22}\dot{\gamma}R + \alpha_{23}\dot{Z}_o}{\alpha_{11}\dot{R} + \alpha_{12}\dot{\gamma}R + \alpha_{13}\dot{Z}_o}\right\} \tag{8}$$

Relative Wind Velocity:

$$V_R = \left\{ \dot{R}^2 + (\dot{\gamma} R)^2 + \dot{Z}_o^2 \right\}^{1/2} \tag{9}$$

Roll Rate:

$$p = (\dot{\psi} + \dot{\gamma})\alpha_{13} + \dot{\Theta} \cos\Phi \tag{10}$$

Pitch Rate:

$$q = (\dot{\psi} + \dot{\gamma})\alpha_{23} - \dot{\Theta} \sin\Phi \tag{11}$$

Yaw Rate:

$$r = (\dot{\psi} + \dot{\gamma})\alpha_{33} + \dot{\Phi} \tag{12}$$

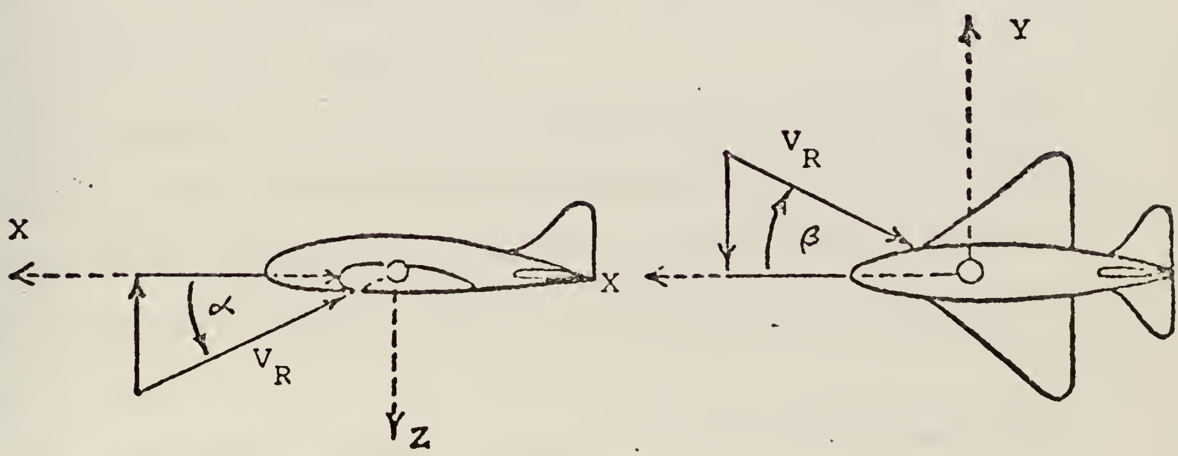

Figure 3

Definition of Positive α and β Angles

E. AERODYNAMIC FORCE AND MOMENT COEFFICIENTS

The general form of the aerodynamic force and moment coefficients employed in the equations of motion are as follows:

$$C_{F_i} = C_{F_{i_1}} + C_{F_{i_2}} f(p) + C_{F_{i_3}} f(q) + C_{F_{i_4}} f(r) + C_{F_{i_5}} f(\delta a) + C_{F_{i_6}} f(\delta r) + C_{F_{i_7}} f(\delta e) \qquad i = x, y, z \tag{13}$$

$$C_{m_i} = C_{m_{i_1}} + C_{m_{i_2}} f(p) + C_{m_{i_3}} f(q) + C_{m_{i_4}} f(r) + C_{m_{i_5}} f(\delta a) + C_{m_{i_6}} f(\delta r) + C_{m_{i_7}} f(\delta e) \qquad i = x, y, z \tag{14}$$

where δa δr δe represent the angular deflections of the ailerons, rubber and elevator respectively.

All of the above coefficients on the right side of the equality sign are aerodynamic force and moment derivatives as defined in Table II. They must be experimentally determined and tabulated for a wide range of α and β vlaues. f(p, q, or r) are functions of roll, pitch and yaw rates.

The coefficients in Table II utilize $\bar{c}$ as a common characteristic length, thus allowing the use of a single length parameter input to the computer program. Since $\dot{\alpha}$ and $\dot{\beta}$ are zero for the steady spin, aerodynamic derivatives with respect to these two parameters have not been included in Table II.

	SUBSCRIPT j						
i=x, y, or z	1	2	3	4	5	6	7
C_{Fij}	$C_{Fi}(\alpha,\beta)$	$\frac{\partial C_{Fi}}{\partial\left(\frac{p\bar{c}}{2V}\right)}$	$\frac{\partial C_{Fi}}{\partial\left(\frac{q\bar{c}}{2V}\right)}$	$\frac{\partial C_{Fi}}{\partial\left(\frac{r\bar{c}}{2V}\right)}$	$\frac{\partial C_{Fi}}{\partial \delta_a}$	$\frac{\partial C_{Fi}}{\partial \delta_r}$	$\frac{\partial C_{Fi}}{\partial \delta_h}$
C_{Mij}	$C_{Mi}(\alpha,\beta)$	$\frac{\partial C_{Mi}}{\partial\left(\frac{p\bar{c}}{2V}\right)}$	$\frac{\partial C_{Mi}}{\partial\left(\frac{q\bar{c}}{2V}\right)}$	$\frac{\partial C_{Mi}}{\partial\left(\frac{r\bar{c}}{2V}\right)}$	$\frac{\partial C_{Mi}}{\partial \delta_a}$	$\frac{\partial C_{Mi}}{\partial \delta_r}$	$\frac{\partial C_{Mi}}{\partial \delta_h}$

Table II

Definition of the Aerodynamic Derivatives[2]

The validity of any spin solution results is dependent upon how well the aerodynamic forces and moments have been modeled. One needs only to visualize the different dynamic environment that exists between the static model in a wind tunnel and the actual airplane experiencing three dimensional stalled flow in a spin. Herein lies the greatest weakness in the analytic spin solution effort. The static wind tunnel data utilized to determine the above 42 coefficients provides, at best, a very crude model of the actual conditions encountered by a spinning aircraft. However, for want of better data, the conventional data was utilized in order to exercise the program and to seek a first-order approximation to a spin solution.

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[2] Aerodynamic derivatives with respect to $\dot{\alpha}$ and $\dot{\beta}$ have not been included since $\dot{\alpha}$ and $\dot{\beta}$ are zero for the steady spin.

F. SOLVING THE EQUATIONS OF MOTION

A casual examination of the equations of motion found in Appendix II reveals that they have the following characteristics:

1. ordinary second order differential equations
2. extensively coupled
3. highly nonlinear
4. force and moment terms are a function of three-dimensional flow over the geometry of the airplane.

The above characteristics preclude direct analytical integration and thus one is forced to seek a numerical solution scheme which will provide the greatest insight into the physical problem and also provide some degree of uniform convergence. Additionally, from numerical considerations the extreme length of the equations dictate the need for a method which maintains numerical significance and does not require excessive computational time.

Two iterative solution methods were selected; the integration method which was utilized for steady and quasi-steady state solutions and the gradient method for the purely steady-state solution efforts. In both instances, the set of equations could be visualized as a "black box" function transform with independent variables as inputs and the equation residuals as outputs.

G. THE INTEGRATION SOLUTION METHOD

The integration method can be visualized as the incremental tracking of the trajectory of a multiple degree of freedom spring-mass-damper system. The incremental changes are generated by "psuedo" integration of the aerodynamic force and moment inbalances manifested in the equation residuals. This set of incremental changes constitutes

the "trajectory". The psuedo integration is accomplished by taking advantage of the form of the equations whereby the residuals are essentially the accelerations of the six spin degrees of freedom. Application of a common increment of time to elementary Euler integration formulae allows the computation of a dynamically consistent set of incremental changes in the independent variables and thus generates new values for the subsequent iteration. Essentially, the method duplicates, in a numerical sense, what the aircraft experiences under actual flight conditions.

One of the major advantages of casting the equations of motion in a cylindrical reference frame, is the ease with which tumbling motion can be separated from the motion associated with a steady state spin. Constant values of Z_0, R, $\dot{\delta}$, Θ, Ψ, and Φ indicate a steady state solution, while non-zero values of $\dot{\Theta}$, $\dot{\Psi}$, or indicate tumbling motion. The computational scheme discards any angular momentum inconsistent with a steady-state spin and thus on each iteration any tumbling motion is suppressed. Such motion, if retained, could possibly carry the solution path (trajectory) through a steady state spin solution.

The iterative procedure used in the computer was basically very simple. In response to a set of initial conditions (Z_0, R, $\dot{\delta}$, Θ, Ψ, Φ) and related constants, the corresponding accelerations were computed. Application of a common time increment (Δt) to the basic Euler integration formulae yields a set of incremental changes which are then added to the previous set of independent variables to obtain a new set of initial conditions. The iteration process continues until the sum of the absolute values of the accelerations, denoted as S, is reduced

to less than some specified value. The "spring-mass-damper" system thus moves closer to equilibrium as it converges to a steady state solution.

The acceleration $\ddot{Z}_0$ and $\ddot{R}$ can be computed directly from the independent and dependent variables. The other accelerations ($\ddot{\gamma}$, $\ddot{\Theta}$, $\ddot{\Psi}$, $\ddot{\Phi}$) however, must be computed by a looping routine since the associated equations are coupled to all of the angular accelerations. The looping ceases once a consistent set of accelerations is obtained.

The incremental changes are obtained using the following equations:

$$\dot{Z}_{0\,(N+1)} = \dot{Z}_{0\,(N)} + (\ddot{Z}_0 \,\Delta t)a \tag{15}$$

$$\dot{R}_{(N+1)} = \dot{R}_{(N)} + (\ddot{R}\,\Delta t)b \tag{16}$$

$$R_{(N+1)} = R_{(N)} + \left(\dot{R}_{(N+1)}\,\Delta t + \ddot{R}\,\frac{\Delta t^2}{2}\right)b \tag{17}$$

$$\dot{\gamma}_{(N+1)} = \dot{\gamma}_{(N)} + (\ddot{\gamma}\,\Delta t)c \tag{18}$$

$$\dot{\Theta}_{(N+1)} = \dot{\Theta}_{(N)} + (\ddot{\Theta}\,\Delta t)d \tag{19}$$

$$\Theta_{(N+1)} = \Theta_{(N)} + \left(\dot{\Theta}_{(N+1)}\,\Delta t + \ddot{\Theta}\,\frac{\Delta t^2}{2}\right)d \tag{20}$$

$$\dot{\Psi}_{(N+1)} = \dot{\Psi}_{(N)} + (\ddot{\Psi}\,\Delta t)e \tag{21}$$

$$\Psi_{(N+1)} = \Psi_{(N)} + \left(\dot{\Psi}_{(N+1)}\,\Delta t + \ddot{\Psi}\,\frac{\Delta t^2}{2}\right)e \tag{22}$$

$$\dot{\Phi}_{(N+1)} = \dot{\Phi}_{(N)} + (\ddot{\Phi}\,\Delta t)f \tag{23}$$

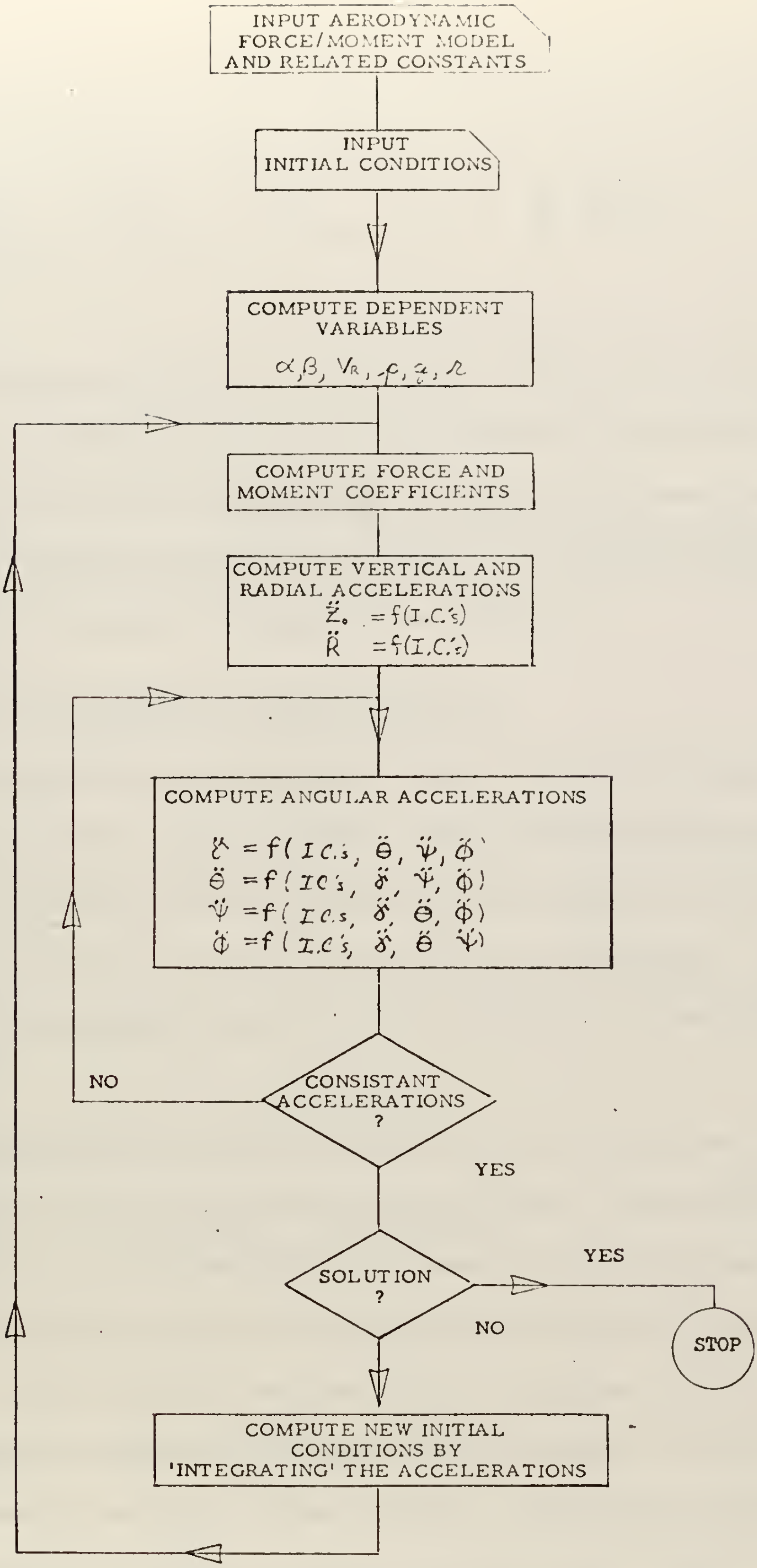

Figure 4

A Schematic Representation of the Integration Solution Method

$$\phi_{(N+1)} = \phi_{(N)} + (\dot{\phi}_{(N+1)} \Delta t + \ddot{\phi} \frac{\Delta t^2}{2}) f \tag{24}$$

The constants (a, b, c, d, e, and f) are solution convergence factors which can be used to weigh the solution "rate" of a particular parameter. Also, one should note that the ($\dot{\theta}$), ($\dot{\psi}$), and ($\dot{\phi}$) expressions are used only in conjunction with the full equations. A schematic representation of the iterative process is depicted in Figure 4.

H. THE GRADIENT SOLUTION METHOD

The existance of a dynamically stable equilibrium solution can be verified utilizing a set of equations which exclude all time dependent terms. A gradient solution method was developed to search for such solutions using the previously discussed short equations. The method utilized is commonly referred to as the optimal method of steepest descent.[4]

Conceptually, the solution space can be visualized as a series of peaks and valleys with solutions being the floor of any valley whose altitude is zero. The method is initiated at some 'geographic' position (initial conditions) where the "altitude" is measured (criterion function value). If the "altitude" were non-zero, the slope (gradient) of the "hill" would be measured and then the independent variables would be incrementally "marched" down the hill. The process continues until the independent variables reach the bottom of the "valley".

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[4]Reference [7], page 275.

Mathematically, the criterion function is defined as the sum of the absolute values of the equation residuals. A solution is obtained when

$$S = \sum_{i=1}^{6} |S_i| = 0 \tag{25}$$

where S_i are the equation residuals computed from the given independent variable vector, X_i (i = 1, 6).

The rapid reduction of S to zero is accomplished by iteratively choosing an increment δx_i such that δS approaches -S where

$$\delta S = \sum_{i=1}^{6} \left(\frac{dS}{dX_i}\right) \delta X_i = -S \tag{26}$$

This is achieved by choosing δx_i as the product of the criterion gradient and a gain constant K;

$$\delta x_i = K \frac{dS}{dX_i} \tag{27}$$

However, by noting that

$$\delta x_i = \sum_{i=1}^{6} \left(\frac{dS}{dX_i}\right)\left(K \frac{dS}{dX_i}\right) = -S \tag{28}$$

or

$$\delta S = K \sum_{i=1}^{6} \left(\frac{dS}{dX_i}\right)^2 = -S \tag{29}$$

K can be expressed as

$$K = \frac{-S}{\sum_{i=1}^{6} \left(\frac{dS}{dX_i}\right)^2} \tag{30}$$

The subsequent value of the i^{th} element of the X vector becomes

$$X_{i(N+1)} = X_{i(N)} + \delta x_{i(N)} \qquad i = 1, 2, .., 6 \tag{31}$$

where the subscript n refers to a sequential iteration number.

S_{n+1} is then computed as a function of X_{n+1}; if the condition

$$S_{N+1} < S_N \tag{32}$$

is satisfied, then the increment vector δx_N is reapplied,

$$X_{N+2} = X_{N+1} + \delta x_N \tag{33}$$

and this 1-D search process is continued in the direction of the original gradient until

$$S_{N+1} > S_N \tag{34}$$

whereupon new values of the gradients are computed. Since the process avoids the computation of the partial derivatives whenever the condition expressed in equation (32) is satisfied, significant amounts of computer computational time is saved and hence, the name 'optimal' gradient solution method is achieved. Figure 5 includes a flow chart of this technique.

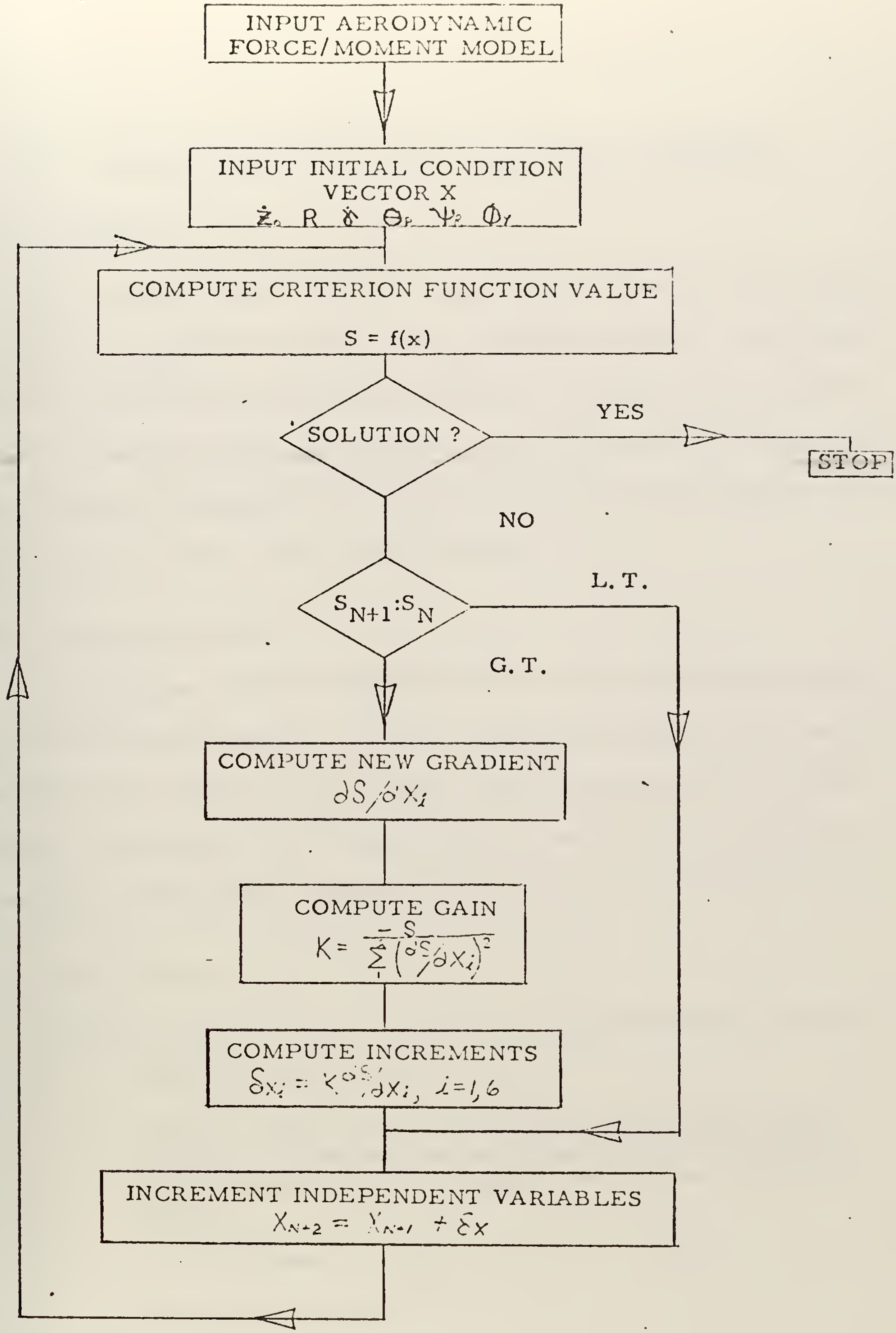


Figure 5

A Schematic Representation of the Gradient Solution Method

III. AIRCRAFT SPIN SOLUTION COMPUTER PROGRAM

A. GENERAL

This section presents briefly a general description of the digital computer program designed to solve the spin equations of motion for dynamically stable equilibrium solutions. A listing of the program as well as specific usage data and flow charts is presented in Appendix II. A program hierarchy is included as Figure 6 and an explanation of the program output is described by Figure 7.

B. PROGRAM USAGE

The program reads the tabular force/moment coefficients and related aircraft constants and then commences to iteratively seek a solution based upon the first initial condition data set that is read. Upon termination of a particular solution, the program restarts by reading the next initial condition data set.

C. INPUT DATA

The user must provide the following information to the program:

1. Type aircraft (F4, F111, etc.)
2. Tabular values of the force and moment coefficients/ stability derivatives (as described in Table II) and the corresponding values of angle of attack and sideslip angle
3. Aircraft mass
4. Atmospheric density
5. Mean aerodynamic chord
6. Gravitational constant
7. Total wing area

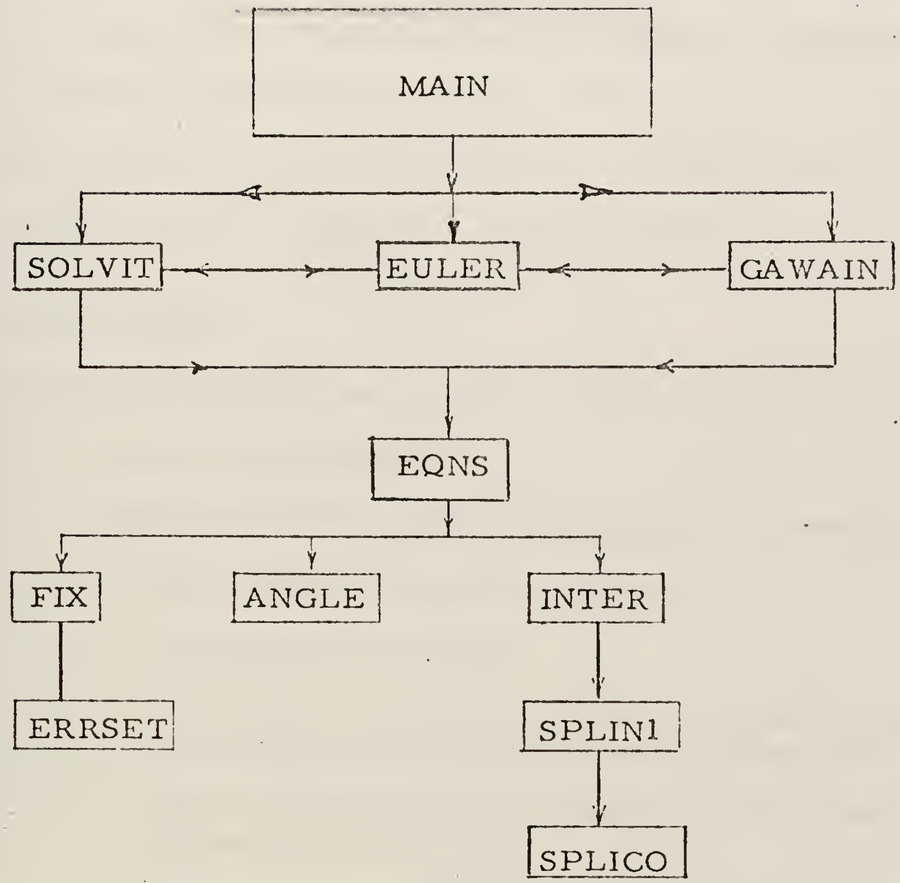


Figure 6

Airplane Spin Solution Program Hierarchy

8. Principal moments of inertia

9. Control deflections

10. Initial conditions of the independent variables

The user also must stipulate the solution method, equation form, solution criteria, time increment, output format and solution convergence factors.

The solution convergence factors available for the integration method, allow the user to accelerate or decelerate the solution "rate" of any of the six independent variables. For example, by equating a particular factor to zero, that variable will remain constant, thus reducing the number of degrees of system freedom.

D. OUTPUT DATA

The printed output format is similar for the integration and gradient methods. The following is printed:

1. Listing of all input data under appropriate headings
2. Iterative results (integration method)
   a. iteration number (M1)
   b. the number of looping iterations required to obtain a consistent set of angular accelerations (M2)
   c. the independent variables with their associated first and second derivatives
   d. relative wind velocity, angle of attack, and sideslip angle
   e. sum of the absolute values of the accelerations
3. Iterative results (gradient method)
   a. iteration number (M1)
   b. the iteration number corresponding to the last gradient computation (M2)

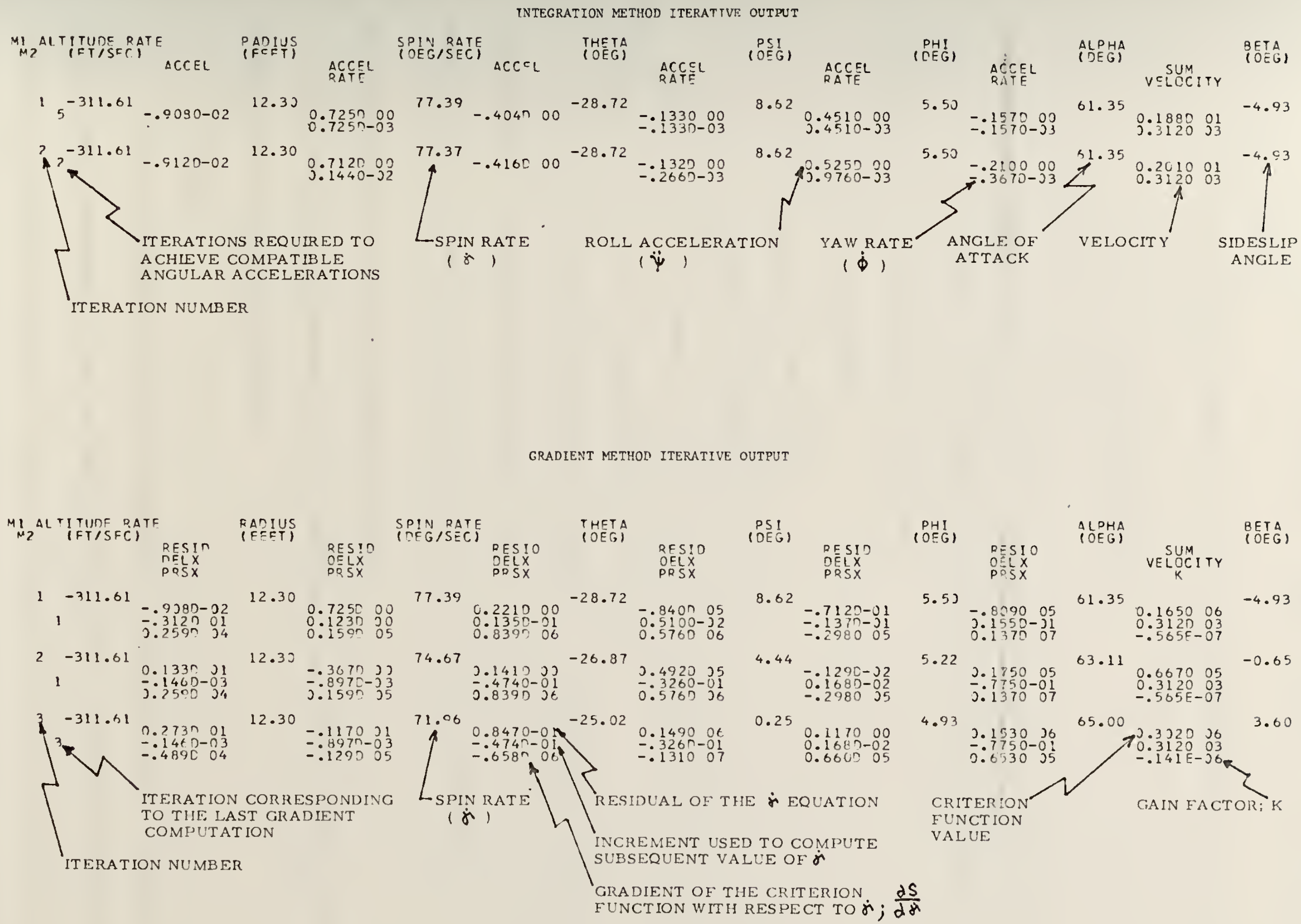


Figure 7

Explanation of the Airplane Spin Solution Printed Output

c. the six independent variables with their corresponding residuals, incremental values, and partial derivatives

d. relative wind velocity, angle of attack, and sideslip angle

e. criterion value; the sum of the absolute values of the residuals

f. K; the solution gain factor

4. Summary output

Upon normal termination of the integration method, the iterative output is summarized. The listing includes the following data corresponding to every twenty-five iterations:

1. iteration number
2. independent variables
3. dependent variables (α, β , V_R)

Also, the above data corresponding to the smallest sum of the absolute values of the accelerations is printed.

In the event of an abnormal termination, the program will print an appropriate message which informs the user as to the reason for the termination as well as the values of the offending variable(s).

E. PROGRAM MODULE DESCRIPTION

The program consists of nine modules; one main and eight subroutines as described below:

1. MAIN

This routine is basically an input/output device. Tabulated data, fixed constants, and initial conditions are read into common memory, angular units are converted from degrees to radians; the optional output format is established and the first page is printed. The optional equation form and solution method, being determined,

a call is then made on the appropriate solution subroutine for subsequent computation.

2. SOLVIT

The integration solution method described in Section II is programmed into this subroutine. The iterative results are printed and appropriate summary data is presented on the last page of the printout.

3. GAWAIN

This subroutine incorporates the optimized gradient method as described in Section II. Its operation is similar to SOLVIT, however, no summary data is printed.

4. EQNS

Subroutine EQNS includes all of the independent and dependent equations. Calls are made primarily from either GAWAIN or SOLVIT. The current set of initial conditions is then used to compute the direction cosines, angle of attack and sideslip angle. Subroutine ANGLE is called to determine whether Alpha and Beta are within the appropriate definition limits. If so, the force and moment coefficients are then determined by interpolating the appropriate matrices using subroutines INTER, SPLIN1, and SPLICO.

The residuals are computed using either the short, modified or long form of the equations. A looping routine is utilized with the latter forms to obtain a consistent set of angular accelerations. Finally, upon computation of the sum of the absolute values of the residuals, execution is returned to the calling program.

5. EULER

The equations of motion require that angular units be in radians and that the reference Euler orientation angles be utilized. Subroutine EULER is used to convert computer variables to the output form of

angular units of degrees and ordered orientation angles. The subroutine is utilized by MAIN, GAWAIN and SOLVIT in order to allow output of the more usable ordered angles.

6. ANGLE

Subroutine ANGLE is utilized primarily by EQNS to check whether alpha or beta has exceeded the upper or lower limits of data definition. If the limits were exceeded, the iterative solution effort would be terminated and control returned to MAIN. ANGLE also reduces the absolute magnitude of angular displacements to less than 2π radians.

7. FIX

Subroutine FIX is used in conjunction with IBM supplied ERRSET to facilitate termination of an iterative solution in the event of an underflow, overflow or divide check while control is in EQNS. Control is returned to MAIN rather than effecting program termination as is the usual case.

8. INTER

Subroutine INTER is utilized by EQNS to interpolate the tabular force and moment coefficient data for specified values of alpha and beta. INTER calls SPLIN1 for the cubic interpolation of each matrix or vector.

9. SPLIN1 and SPLICO

Subroutines SPLIN1 and SPLICO were written by Mr. Rod Kure and are maintained by the NPS Computer Center Staff. Their primary function is to provide cubic interpolation of a given matrix or vector.

V. AIRPLANE SPIN EQUATION VALIDATION

The airplane spin equations of motion were validated by establishing initial conditions for a power-off glide configuration and then applying them to the equations using the spin solution program.

A small computer program, TRIM, was prepared to enable the selection of a set of dynamically stable initial conditions. The TRIM program flowchart, the program listing and the sample output is included in Appendix D.

The condition of a power-off glide is mathematically described by three expressions:

$$TAN\,(\alpha-\theta) = C_D/C_L \tag{34}$$

$$W = \tfrac{1}{2}\rho V^2 S\left\{C_L\,COS\,(\alpha-\theta) + C_D\,SIN\,(\alpha-\theta)\right\} \tag{35}$$

$$C_m = \frac{-C_{m_{\delta e}}}{\delta e} \tag{36}$$

where w, α, θ, δe, C_l, C_d, and C_m are as depicted in Figure 8. The values for C_l and C_d are determined by transposing from the principal to the stability axis systems as indicated below:

$$C_L = -(C_z + C_{z_{\delta e}}\,\delta e)\,Cos\,\alpha + (C_x + C_{x_{\delta e}}\,\delta e)\,Sin\,\alpha \tag{37}$$

$$C_D = -(C_x + C_{x_{\delta e}}\,\delta e)\,Cos\,\alpha + (C_z + C_{z_{\delta e}}\,\delta e)\,Sin\,\alpha \tag{38}$$

Given constant values for α, w, ρ, and S, one can solve equations (34)--(36) for the quantities δe, θ, and V. Subsequently the variables

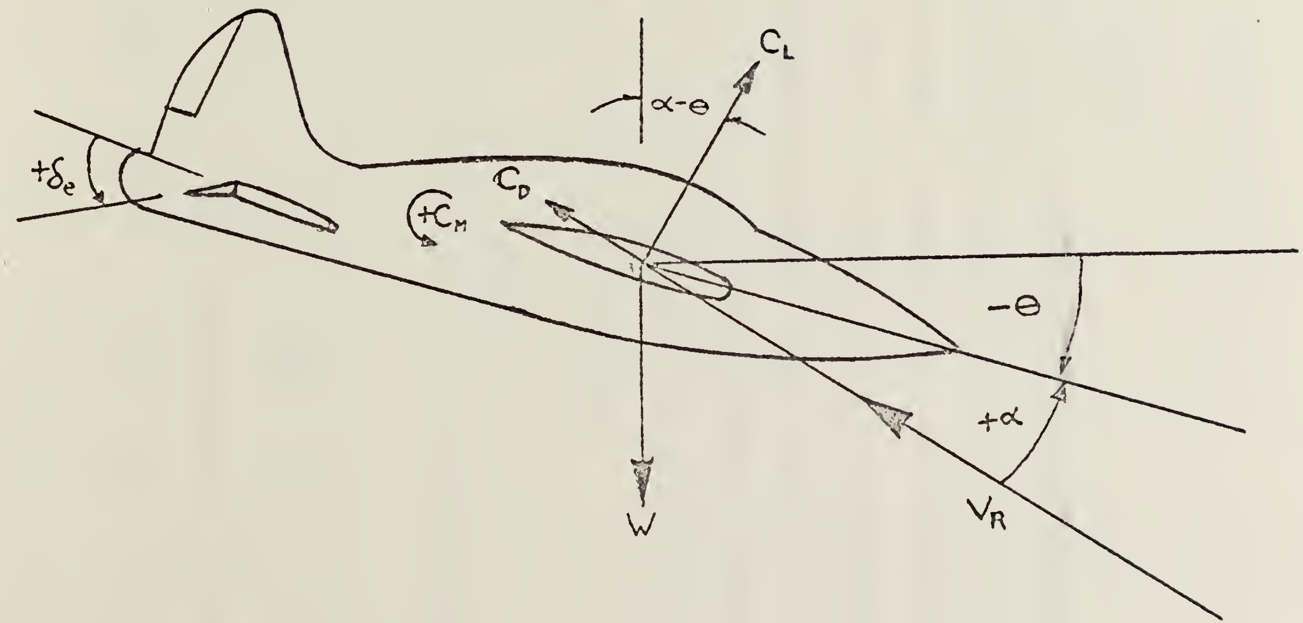


Figure 8

Schematic of the Aerodynamic Parameters for an Airplane in a Power-off Glide

$$\dot{Z}_o = V \sin(\alpha - \theta) \quad (39)$$

$$\dot{R} = V \cos(\alpha - \theta) \quad (40)$$

can then be determined.

The variables Z_o, R, and δe are thus established independent of the spin equations of motion but based upon the same aerodynamic force and moment model utilized by the spin solution program. Application of these initial conditions to the spin solution program containing Buehler's spin equations resulted in an immediate convergence whereby the sum of the absolute values of accelerations (equation residuals) was equal to 0.0182. Table III provides a listing of the pertinent data utilized to achieve the equation validation.

Table III

Pertinent Data Relating to Equation Validation

	Z_o	R	θ_p	α	V_r	δe
TRIM Input:	F-111 force and moment model; air density = $.1553 \times 10^{-2}$					
TRIM Output	77.01[1]	-381.1[1]	.5763[1]	12.0	-388.8	-5.534[1]
Integration Method Solution[2]	76.99	-381.0	.57	12.0	389.0	

Notes: 1. Spin solution program input as per Table VIII
2. SUM = .0188

The spin solution program requires special care when exercising it for straight and level flight. Due to EULER restrictions about accepting quadrant II and III and cardinal orientation angles, the initial condition values should be applied as in Table IV below.

TABLE IV

Initial Conditions for Equation Validation

VARIABLE	VALUE
ρ, S, W	As used in TRIM Program
$\delta_e, \dot{Z}_o, \Theta, \dot{R}$	As obtained from TRIM Output
$\delta_a, \delta_r, \dot{\delta}_r$	0. 0
Ψ, Φ	. 0001
NSFLAG	0

The results of the equation validation computer tests conclusively validate the spin equations of motion for the power-off glide configuration. This procedure can only be considered, at best, qualitative, for it does not test the equations in a time dependent environment and configuration. Complete validation would require the generation of time histories and subsequent comparison to base line data such as is included in Reference [2].

V. NUMERICAL ANALYSIS AND CONCLUSIONS

A. AERODYNAMIC FORCE AND MOMENT MODELS

The primary purpose of this thesis effort was to develop an aircraft spin solution computer program. During the program development, there were no idealized data available with known spin solutions, therefore the author utilized actual static wind tunnel data to provide an aerodynamic force and moment model. This was done in one case with the objective of ultimately comparing actual aircraft spin performance with computer generated predictions.

Two models were utilized. Tabular F-111 data was obtained from Reference [3] and F-4b data were extracted from curves presented in Reference [4]. In both cases appropriate values were multiplied by b/c in order to normalize to the common dimension utilized in the computer program, i.e., the mean aerodynamic chord. The above data are included in Appendix E.

B. SOLUTION METHOD EVALUATION

1. Integration Solution Method

The integration solution method was the primary vehicle utilized to develop the spin equations of motion. As discussed in Section II, the primary advantage lies with the characteristic of the equation residuals being equivalent to the appropriate acceleration terms.

The method must be initiated with a set of initial conditions. Many sets of various initial conditions were utilized during the development but for the sake of brevity only the results from one arbitrary set will be discussed.

It was evident early in the development phase that the modified equations provided the greatest potential. The full equations were suitable only for verifying the stability of solutions previously obtained from the modified equations. When the full equations were used to evaluate a set of arbitrary initial conditions, the angular orientation rates rapidly increased to disproportionate values and induced tumbling motion which, in turn, resulted in a divergent solution. The modified equations, however, enable quasi-steady solutions and are responsive to small changes in the time increment and control deflections.

The solution paths generated by the integration method were characterized by an initially rapid convergence to a minimum value of the criterion function. The orientation values reorient to a nearly optimal configuration; the sink rate seeks the acceleration minimum; and the spin rate and radius effect trade-offs to achieve a consistent velocity value. Occasionally, the solution path became divergent in that the program was unable to find a consistent set of coupled accelerations. This was evidenced by the printout of the number of iterations required by the acceleration "do loop" within Subroutine EQNS. The "do loop" limit is set at twenty iterations to minimize computer time, however, only two iterations are usually needed for a stable solution path. Divergence is characterized by large and erratic changes in the independent variables such that alpha or beta exceed their respective fields of definition, thus resulting in solution termination. Solution path divergence can be minimized through use of a small time increment. The author determined that a range between .001 and .05 seconds proved optimal. The solution convergence factors were not utilized sufficiently to merit discussion.

Once the initial convergence was achieved, the individual solution paths seemed to vary in a seemingly arbitrary manner. While one variable tended toward a minimum acceleration value, other variables were producing increasing values. The variables continued to "wander"; changing magnitude and sign until either computer time was exhausted or the alpha or beta limits were exceeded.

All solution efforts using F-4 data with pro-spin control deflections resulted in solution paths which exceeded the sideslip angle limits. Inclusion of rotary balance data to compensate for differential sideslip angles along the fuselage might inhibit such yawing action.

Utilization of the integration method requires that the user learn to iterate the output to optimize the achievement of a final solution. This process is as described below:

a. An arbitrary set of initial conditions is applied to the program to initiate a 500 iteration solution effort. This set of initial conditions should be such so as to approximate the anticipated spin solution mode.

b. The initial condition set which yields the best convergence is modified by substituting the value for $\dot{Z}_o$ corresponding to the minimum $\ddot{Z}_o$ in the 500 iteration output. An alternative is to substitute the set of three orientation angles corresponding to the minimum absolute sum of the orientation angle accelerations. Similarly, R and $\dot{\gamma}$ can be substituted. The resultant set of revised initial conditions is then used to initiate subsequent solution efforts of shorter (150 iterations) duration.

c. The process is continued until a best solution is obtained.

d. The final solution is tested for stability by using the full equations with a small time increment.

Figure 9 and Table V illustrate a typical solution effort using an arbitrary set of initial conditions.

The integration method results lead to a number of conclusions:

a. The F-111 aerodynamic force and moment model precluded finding stable equilibrium solutions. This can be attributed to any of four reasons: (1) there were no truly steady state solutions; only oscillatory modes exist; (2) the "right" set of initial conditions was never employed and therefore a steady state solution path was not intercepted; (3) the steady state modes were lightly damped and therefore, may be very slow in convergence; the program wasn't exercised long enough to find the solution; and (4) the integration solution method itself precludes uniform convergence of all degrees of freedom. This would be due to self-excitation of the non-linear coupled acceleration terms.

b. The F-4 data do not model the actual aircraft since all initial conditions yielded solution paths with divergent yaw rates. The actual aircraft does not exhibit this trait and, in fact, demonstrates several steady and quasi-steady spin modes.

c. The method is cumbersome to use because it requires that the user iteratively adjust inputed initial conditions in order to achieve a solution.

d. The radius rate terms incorporated in the modified equations enables the determination of quasi-steady state solutions using the F-111 data.

e. The orientation rate terms in the full equations were not compatible with the course integration techniques of this method.

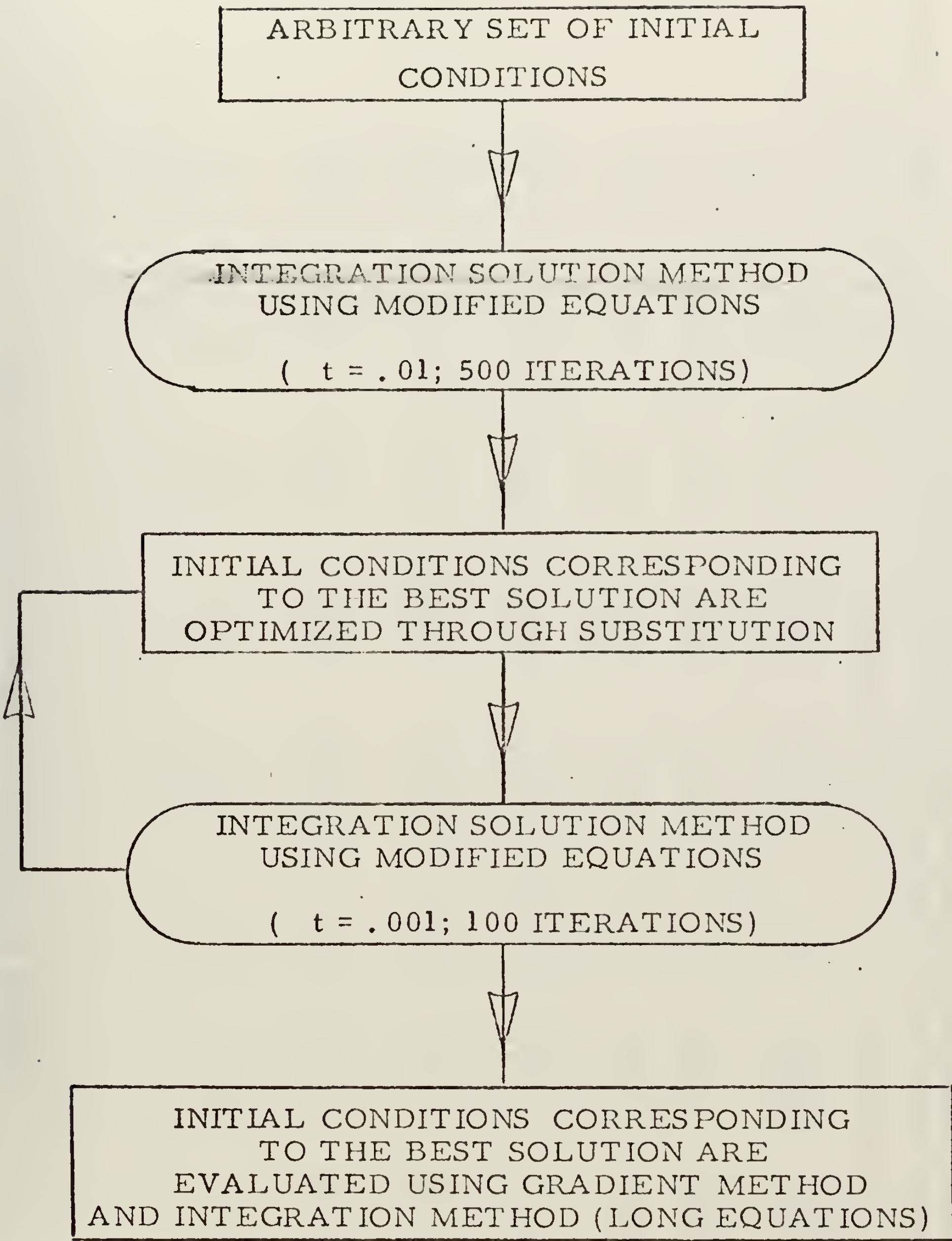


Figure 9

Schematic of a Typical Integration Solution Effort

TABLE V

Results of a Typical Integration Solution Effort

RUN #	$\dot{Z}_o$	R	$\dot{R}$	$\dot{\gamma}$	θ_p	ψ_R	Φ_Y	SUM
1	-400.0	50.0	0.0	30.0	-30.0	10.0	-5.0	--
	-372.36	21.37	31.8	67.68	-29.38	9.56	-1.22	25.41
2	-332.36	21.37	0.0	67.68	-29.38	9.56	-1.22	--
	-329.98	12.30	-11.7	79.13	-28.90	8.93	3.74	7.89
3	-317.33	12.30	0.0	79.13	-28.90	8.93	3.74	--
	-317.26	12.30	.063	77.87	-28.88	8.94	3.78	4.61
4	-317.26	12.30	0.0	77.87	-28.72	8.62	5.50	--
	-317.25	12.30	.0027	77.63	-28.72	8.62	5.50	2.53
5	-314.61	12.30	0.0	77.39	-28.72	8.62	5.50	--
	-314.61	12.30	.003	77.39	-28.72	8.62	5.50	2.06
6	-311.61	12.30	0.0	77.39	-28.72	8.62	5.50	--
	-311.61	12.30	.0003	77.39	-28.72	8.62	5.50	1.88
7[1]	-311.61	12.30	n/a	77.39	-28.72	8.62	5.50	--
	-311.61	12.30	n/a	68.99	-35.30	14.58	6.15	29.2×10^4

Upper values correspond to the inputted initial conditions while the lower values correspond to the best solution (minimum value of the criterion function: SUM).

NOTES:
1. Run #7 utilized the gradient solution method.
2. The inputted initial conditions correspoinding to Run #6 were also applied to the long equations using the integration method but the solution path was diverging when computation was terminated.
3. F-111 aerodynamic model data was utilized with full pro-spin control deflections.

f. The computer program needs to be modified to incorporate a subroutine which will compute the spin radius as well as the cylindrical inertial radius as in the present version.

2. Gradient Solution Method

The gradient solution method and its derivatives from optimal control theory have, perhaps, the greatest potential in the area of steady-state spin prediction. The method programmed by the author can only be considered crude when compared to some of the more sophisticated methods which utilize hybrid applications.

The results obtained from a typical F-111 gradient solution effort provide the basis for evaluation of this method as well as yield insight into the F-111 solution space. Table VI includes a summary of these results. One should note that there was an initial convergence of the angular variables in the first fifty iterations. After iteration #75, the solution stabilized but the criterion value alternated between 27.0 and 102.0; this was a result of the alternating residuals of the pitch and yaw equations. The sensitivity of the solution is extreme when one considers that a 0.4% change in the absolute sum of the independent variables results in a 2,740% reduction in the absolute sum of the acceleration. Specifically, the alternating changes in the Θ and Φ residuals are the result of incremental changes in the Θ and Φ values of only 10^{-4} radians!

The solution convergence becomes static due to the characteristically large gradients (10^3-10^7) and resultant small incremental step sizes. All gradient solutions attempted, resulted in no significant change in the values for $\dot{Z}_o$ and R parameters.

Two conclusions can be made from the gradient solution method results:

TABLE VI

Summary of a F-111 Gradient Solution Effort

	$\dot{Z}_o$	R	γ	θ_P	ψ_R	ϕ_Y	SUM
Iteration Number			Iterative Results				
0	-200.0	100.0	10.0	-31.0	5.0	-5.0	7.38×10^3
25	-200.0	100.0	9.42	-31.5	5.35	-5.08	2.63×10^3
50	-200.0	100.0	9.39	-31.38	5.34	-5.05	0.73×10^2
75	-200.0	100.0	9.39	-31.37	5.34	-5.05	1.02×10^2
76	-200.0	100.0	9.39	-31.37	5.34	-5.05	$.279 \times 10^2$
100	-200.0	100.0	9.39	-31.37	5.34	-5.05	1.02×10^2
101	-200.0	100.0	9.39	-31.37	5.34	-5.05	$.270 \times 10^2$

Other Representative Values as per the Indicated Equations[1]

	$\dot{Z}_o$	R	γ	θ	ψ	Φ
Residual (S_i)	-1.9×10^4	-6.79	0.24	.34[2]	.186	-.39[3]
Increment (δ_{x_i})	3.0×10^{-8}	0.5×10^{-7}	3.7×10^{-5}	$\pm 5.0 \times 10^{-4}$	-1.0×10^{-6}	$\pm 3.5 \times 10^{-5}$
Gradient $\left(\frac{\partial S}{\partial X_i}\right)$	6.1×10^2	-1.0×10^3	$-.74 \times 10^6$	1.0×10^8	2.0×10^6	$.71 \times 10^6$

NOTES:
1. Data is representative from iteration #25 to termination at #350.
2. The θ residual alternated between 0.24 and $\pm$ 61.2.
3. The Φ residual alternated between 0.39 and $\mp$ 14.8.

a. The method was unable to find a steady state solution; however, in one case, it was able to effect a reduction of the criterion function value by four orders of magnitude.

b. The F-111 solution path is highly sensitive to small changes in the independent variable values.

C. ANALYSIS OF THE F-111 AND F-4 STEADY-STATE SOLUTION SPACES

The integration and gradient solution methods were both used to analyze the F-111 and F-4 steady state solution spaces. Initial conditions were generated by a 'Criterion Function Search Program'. This program evaluated criterion function values from a course grid of 186,624 sets of initial conditions for each aircraft.

One can visualize the solution space as a geographical relief with the 'solutions' occurring at the valley floors of zero elevation. The search program essentially measured the 'altitudes' (criterion function values) in this area and found only a very few points whose elevations were less than 10^4. Specifically, in the case of the F-4, only eleven sets of initial conditions generated criterion function values less than 10^4 and only one set of F-111 initial conditions met this criteria. The numerical results of the criterion function search program are included in Table VII.

These generated initial conditions were evaluated using the gradient and integration solution methods. The results are extremely significant because they reveal that it is highly unlikely that a dynamically stable equilibrium solution exists. These solution results are summarized in Figure 10 and Table VIII.

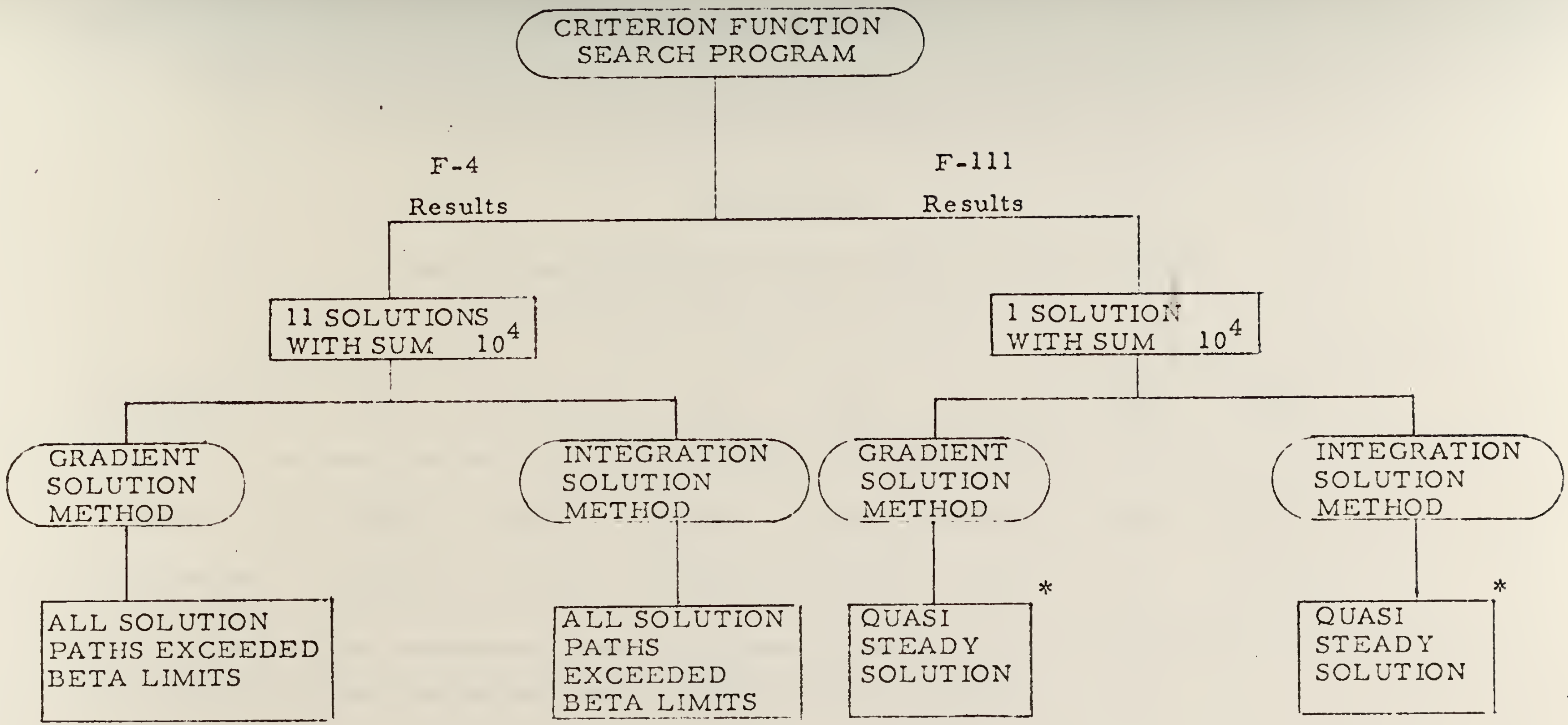
The two conclusions which can be made are:

TABLE VII

Initial Conditions Generated by the Criterion Search Program
Which Yielded Criterion Values Less Than 10^4

A/C	$\dot{Z}_o$	R	$\dot{\delta}$	Θ_P	Ψ_R	Φ_Y	α	β	SUM*
F-111	-200.0	100.0	10.0	-31.0	5.0	-5.0	59.48	-22.42	.738
F-4	-150.0	25.0	10.0	-41.0	5.0	-35.0	44.42	-39.84	.377
F-4	-150.-	25.0	10.0	-51.0	25.0	-15.0	43.91	-38.81	.816
F-4	-150.0	25.0	20.0	-31.0	25.0	- 5.0	64.18	-39.44	.842
F-4	-150.-	25.0	30.0	-21.0	5.0	- 5.0	69.64	-30.16	.640
F-4	-150.0	100.0	10.0	-31.0	15.0	- 5.0	62.03	-35.31	.834
F-4	-150.0	175.0	10.0	-31.0	5.0	- 5.0	59.69	-33.32	.958
F-4	-150.0	175.0	10.0	-41.0	15.0	- 5.0	52.46	-37.27	.761
F-4	-200.0	25.0	10.0	-41.0	5.0	-35.0	44.60	-39.49	.705
F-4	-200.0	175.0	10.0	-41.0	5.0	-25.0	45.27	-39.88	.559
F-4	-300.0	250.0	10.0	-41.0	5.0	-25.0	45.38	-39.49	.513
F-4	-350.0	175.0	30.0	-31.0	5.0	- 5.0	59.79	-38.08	.332

*All values x 10^4



*Refer to Table IV for results.

Figure 10

Criterion Function Search Program Results

TABLE VIII

Gradient and Integration Results from Criterion Search Generated Initial Conditions (F-111)

METHOD	$\dot{Z}_o$	R	δ	θ_P	ψ_R	ϕ_Y	SUM
Gradient	-200.0	100.0	9.39	-31.37	5.34	5.05	27.02
Integration	-258.9	35.46	37.55	-30.84	4.09	-1.51	14.38

The above solutions correspond to the minimum value of SUM.

1. The static wind tunnel data utilized to form the basis for the F-111 and F-4 solution spaces are of such a nature as to preclude finding a dynamically stable equilibrium solution. There was no solution convergence to a criterion value of less than 25.0.

2. The aerodynamic data utilized do not adequately describe the actual aerodynamic force and moment field experienced by the actual aircraft. This is evidenced by the failure to find even a quasi-steady solution similar to flight test results.

The above implications are not surprising considering that the aerodynamics of the spin problem involve low speed, three-dimensional stalled flow and, as such, forces and moments are nonlinear. Thus the superposition of the separate effects of triaxial rotation, control deflection, and other static wind tunnel data cannot be completely justified analytically. The only wind tunnel test technique that presently appears to offer any possibility of yielding satisfactory aerodynamic data for the spin is the rotary balance force and moment measuring apparatus which is described in Reference [6].

VI. SUMMARY OF THE MAJOR CONCLUSIONS

The criterion function search revealed that, for the F-111 and F-4 aerodynamic force and moment models utilized in this study, it was highly improbable that a dynamically stable equilibrium solution existed.

The airplane spin equations described in Reference [1] were qualitatively validated. Quantitative verification is needed and it is recommended that the equations be cast in a form compatible with a time history analysis in order that the resultant data may be compared with the base-line spin data included in Reference [2].

The integration solution method is too cumbersome to be effectively utilized although it can provide information relating to quasi-steady solutions once the user has learned to iteratively manipulate the initial conditions.

The gradient solution method demonstrated its potential by effecting significant reductions in criterion function values to quasi-steady state solutions.

APPENDIX A

FULL SPIN EQUATIONS

The following equations constitute the full equations of motion for a spinning airplane as written in a cylindrical coordinate system.

Z Equation

$$m\ddot{Z}_o + mg = -\left(C_{F_x}\alpha_{13} + C_{F_y}\alpha_{23} + C_{F_z}\alpha_{33}\right)\cdot$$

$$\frac{\rho S}{2}\left\{(\dot{\gamma}R)^2 + (\dot{R})^2 + (\dot{Z}_o)^2\right\}$$

R Equation

$$m\ddot{R} - m\dot{\gamma}^2 R = -\left(C_{F_x}\alpha_{11} + C_{F_y}\alpha_{21} + C_{F_z}\alpha_{31}\right)\cdot$$

$$\frac{\rho S}{2}\left\{(\dot{\gamma}R)^2 + (\dot{R})^2 + (\dot{Z}_o)^2\right\}$$

γ Equation

$$\ddot{\gamma} = \left\{ mR^2 + I_x \alpha_{13}^2 + I_y \alpha_{23}^2 + I_z \alpha_{33}^2 \right\}^{-1} \cdot$$

$$\Big[-2 \left\{ m \dot{\gamma} R \dot{R} + (\dot{\psi} + \dot{\gamma}) \left[(I_x - I_y) \cos\phi \, \alpha_{13} \dot{\phi} \right. \right.$$

$$\left. \left. + (I_x \sin^2\phi + I_y \cos^2\phi - I_z) \alpha_{33} \dot{\theta} \right] \sin\theta \right\}$$

$$- \ddot{\psi} (I_x \alpha_{13}^2 + I_y \alpha_{23}^2 + I_z \alpha_{33}^2)$$

$$- \ddot{\theta} (I_x - I_y) \sin\phi \cos\phi \cos\theta \, \dot{\theta}^2$$

$$+ \left\{ \bar{c} \, (C_{m_x} \alpha_{13} + C_{m_y} \alpha_{23} + C_{m_z} \alpha_{33}) \right.$$

$$\left. - R \, (C_{F_x} \alpha_{12} + C_{F_y} \alpha_{22} + C_{F_z} \alpha_{32}) \right\} \cdot$$

$$\frac{\rho S}{2} \left\{ (\dot{\gamma} R)^2 + (\dot{R})^2 + (\dot{Z}_o)^2 \right\} \Big]$$

θ Equation

$$\ddot{\theta} = \left\{ I_x \cos^2\phi + I_y \sin^2\phi \right\}^{-1} \Big[(\dot{\psi} + \dot{\gamma}) \left\{ I_x \sin^2\phi \right.$$

$$\left. + I_y \cos^2\phi - I_z \right\} \left\{ \dot{\phi} + (\dot{\psi} + \dot{\gamma}) \cos\theta \right\} \sin\theta$$

$$+ 2 (I_x - I_y)(\ddot{\gamma} + \ddot{\psi}) \sin\theta \sin\phi \cos\phi$$

$$+ \left\{ C_{m_x} \cos\phi - C_{m_y} \sin\phi \right\} \frac{\rho S \bar{c}}{2} \left\{ (\dot{\gamma} R)^2 \right.$$

$$\left. + (\dot{R})^2 + (\dot{Z}_o)^2 \right\} \Big]$$

ψ Equation

$$\ddot{\psi} = \left\{ I_x \alpha_{13}^2 + I_y \alpha_{23}^2 + I_z \alpha_{33}^2 \right\}^{-1} \Big[-2(\dot{\psi} + \dot{\gamma}) \cdot$$

$$\Big\{ (I_x - I_y)\alpha_{13}\alpha_{23}\dot{\phi} + (I_x \sin^2\phi + I_y \cos^2\phi$$

$$- I_z) \sin\theta \cos\theta \dot{\theta} \Big\} - (I_x - I_y)\ddot{\theta}\alpha_{13}\cos\phi$$

$$- I_z \ddot{\phi} \alpha_{33} - \ddot{\gamma}(I_x \alpha_{13}^2 + I_y \alpha_{23}^2 + I_z \alpha_{33}^2)$$

$$+ \Big\{ (I_x - I_y)(\sin^2\phi - \cos^2\phi) + I_z \Big\} \sin\theta \dot{\phi} \dot{\theta}$$

$$- (I_x - I_y) \sin\phi \cos\phi \cos\theta \dot{\theta}^2$$

$$+ \Big\{ C_{m_x}\alpha_{13} + C_{m_y}\alpha_{23} + C_{m_z}\alpha_{33} \Big\} \cdot$$

$$\frac{\rho S \bar{c}}{2} \Big\{ (\dot{\gamma} R)^2 + (\dot{R})^2 + (\dot{Z}_0)^2 \Big\} \Big]$$

ϕ Equation

$$\ddot{\phi} = I_z^{-1} \Big[-I_z(\ddot{\psi} + \ddot{\gamma})\alpha_{33} + (I_x - I_y)\Big\{ (\dot{\psi} + \dot{\gamma})\alpha_{13}$$

$$+ \dot{\theta}\cos\phi \Big\} \Big\{ (\dot{\psi} + \dot{\gamma})\alpha_{23} - \dot{\theta}\sin\phi \Big\}$$

$$+ (C_{m_z}) \frac{\rho S \bar{c}}{2} \Big\{ (\dot{\gamma} R)^2 + (\dot{R})^2 + (\dot{Z}_0)^2 \Big\} \Big]$$

MODIFIED EQUATIONS

Z_o Equation:

$$\ddot{Z}_o = -g - (C_{F_X}\alpha_{13} + C_{F_Y}\alpha_{23} + C_{F_Z}\alpha_{33}) \cdot$$

$$\frac{\rho S}{2M}\left\{(\dot{\gamma}R)^2 + (\dot{R})^2 + (\dot{Z}_o)^2\right\}$$

R Equation:

$$\ddot{R} = \dot{\gamma}^2 R - (C_{F_X}\alpha_{11} + C_{F_Y}\alpha_{21} + C_{F_Z}\alpha_{31}) \cdot$$

$$\frac{\rho S}{2M}\left\{(\dot{\gamma}R)^2 + (\dot{R})^2 + (\dot{Z}_o)^2\right\}$$

γ Equation:

$$\ddot{\gamma} = \left\{MR^2 + I_X\alpha_{13}^2 + I_Y\alpha_{23}^2 + I_Z\alpha_{33}^2\right\}^{-1}\Big[-I_X\alpha_{13}^2$$

$$-(I_X - I_Y)\ddot{\theta}\sin\theta\sin\phi\cos\phi - I_Z\dot{\phi}'\alpha_{33}$$

$$-2MR\dot{R}\dot{\gamma} + \Big\{\bar{c}\,(C_{m_X}\alpha_{13} + C_{m_Y}\alpha_{23} + C_{m_Z}\alpha_{33})$$

$$-R(C_{F_X}\alpha_{13} + C_{F_Y}\alpha_{22} + C_{F_Z}\alpha_{32})\Big\} \cdot$$

$$\frac{\rho S}{2}\left\{(\dot{\gamma}R)^2 + (\dot{R})^2 + (\dot{Z}_o)^2\right\}$$

Θ Equation:

$$\ddot{\Theta} = \left\{ I_X \cos^2\phi + I_Y \sin^2\phi \right\}^{-1} \Big[-(I_X - I_Y)(\ddot{\psi} + \ddot{\gamma}_1)\sin\phi \sin\Theta \sin\phi$$

$$+ (I_X \sin^2\phi + I_Y \cos^2\phi - I_Z)\dot{\gamma}_1^2 \sin\Theta \cos\Theta + \left\{ C_{m_X} \cos\phi\right.$$

$$\left. - C_{m_Y} \sin\phi \right\} \frac{\rho S \bar{c}}{2} \left\{ (\dot{\gamma}_1 R)^2 + (\dot{R})^2 + (\dot{Z}_0)^2 \right\} \Big]$$

ψ Equation:

$$\ddot{\psi} = \left\{ I_X \alpha_{13}^2 + I_Y \alpha_{23}^2 + I_Z \alpha_{33}^2 \right\}^{-1} \Big[-(I_X \alpha_{13}^2 + I_Y \alpha_{23}^2$$

$$+ I_Z \alpha_{33}^2)\ddot{\gamma}_1 - (I_X - I_Y)\ddot{\Theta} \sin\Theta \sin\phi \cos\phi$$

$$- I_Z \ddot{\phi} \cos\Theta + \left\{ C_{m_X} \alpha_{13} + C_{m_Y} \alpha_{23} + C_{m_Z} \alpha_{33} \right\} \cdot$$

$$\frac{\rho S \bar{c}}{2} \left\{ (\dot{\gamma}_1 R)^2 + (\dot{R})^2 + (\dot{Z}_0)^2 \right\} \Big]$$

Φ Equation:

$$\ddot{\phi} = \left(\frac{I_X - I_Y}{I_Z} \right) \dot{\gamma}_1^2 \sin^2\Theta \sin\phi \cos\phi - (\ddot{\gamma}_1 + \ddot{\psi}) \cos\Theta$$

$$+ C_{m_Z} \frac{\rho S \bar{c}}{2 I_Z} \left\{ (\dot{\gamma}_1 R)^2 + (\dot{R})^2 + (\ddot{Z}_0)^2 \right\}.$$

SHORT EQUATIONS

Z_o Equation:

$$2gm + \rho S\left\{(\dot{\gamma}R)^2 + (\dot{Z}_o)^2\right\}\left\{C_{F_X}\alpha_{13} + C_{F_Y}\alpha_{23} + C_{F_Z}\alpha_{33}\right\} = 0$$

R Equation:

$$2\dot{\gamma}^2 Rm - \rho S\left\{(\dot{\gamma}R)^2 + (\dot{Z}_o)^2\right\}\left\{C_{F_X}\alpha_{11} + C_{F_Y}\alpha_{21} + C_{F_Z}\alpha_{31}\right\} = 0$$

γ Equation:

$$C_{F_X}\alpha_{12} + C_{F_Y}\alpha_{22} + C_{F_Z}\alpha_{32} = 0$$

θ Equation:

$$2(I_X \sin^2\phi + I_Y \cos^2\phi - I_Z)\dot{\gamma}^2 \sin\theta \cos\theta$$

$$+ \rho S\bar{c}\left\{(\dot{\gamma}R)^2 + \dot{Z}_o^2\right\}\left\{C_{m_X}\cos\phi - C_{m_Y}\sin\phi\right\} = 0$$

ψ Equation:

$$C_{m_X}\alpha_{13} + C_{m_Y}\alpha_{23} + C_{m_Z}\alpha_{33} = 0$$

ϕ Equation:

$$2(I_X - I_Y)\dot{\gamma} \sin^2\theta \sin\phi \cos\phi$$

$$+ \rho S\bar{c}\left\{(\dot{\gamma}R)^2 + \dot{Z}_o^2\right\} C_{m_Z} = 0$$

APPENDIX B

AIRCRAFT SPIN SOLUTION PROGRAM

A. GENERAL

This appendix is intended for the Fortran programmer who is a potential user of the program. The included source listing contains an extensive introductory comment section with which the reader should familiarize himself prior to reading the other sections of this appendix.

B. DATA INPUT

The composition of the data deck is described in the source listing and also depicted in Figure B-1. One should particularly note the composition of data sets #2--#43. The first card of each of the aforementioned sets "tells" the program which tabular data is to follow, its literal name, and whether the data set is a matrix, a vector, or a null set. The subsequent cards of each data set contain the tabular data except where no data is available (null set); in which case, the data set consists only of the first card. Each force/moment coefficient matrix is characterized by a unique integer assignment (NA) as specified in Table B-I. NA is the first value of each of the 42 coefficient data sets. There must be exactly 42 "NA" cards however there is no restriction as to their sequence.

Each solution effort is initiated based upon the initial conditions and associated program variables provided by data set #45. An option is provided (integration method only) for the program variables to remain constant for subsequent solution efforts; only the initial

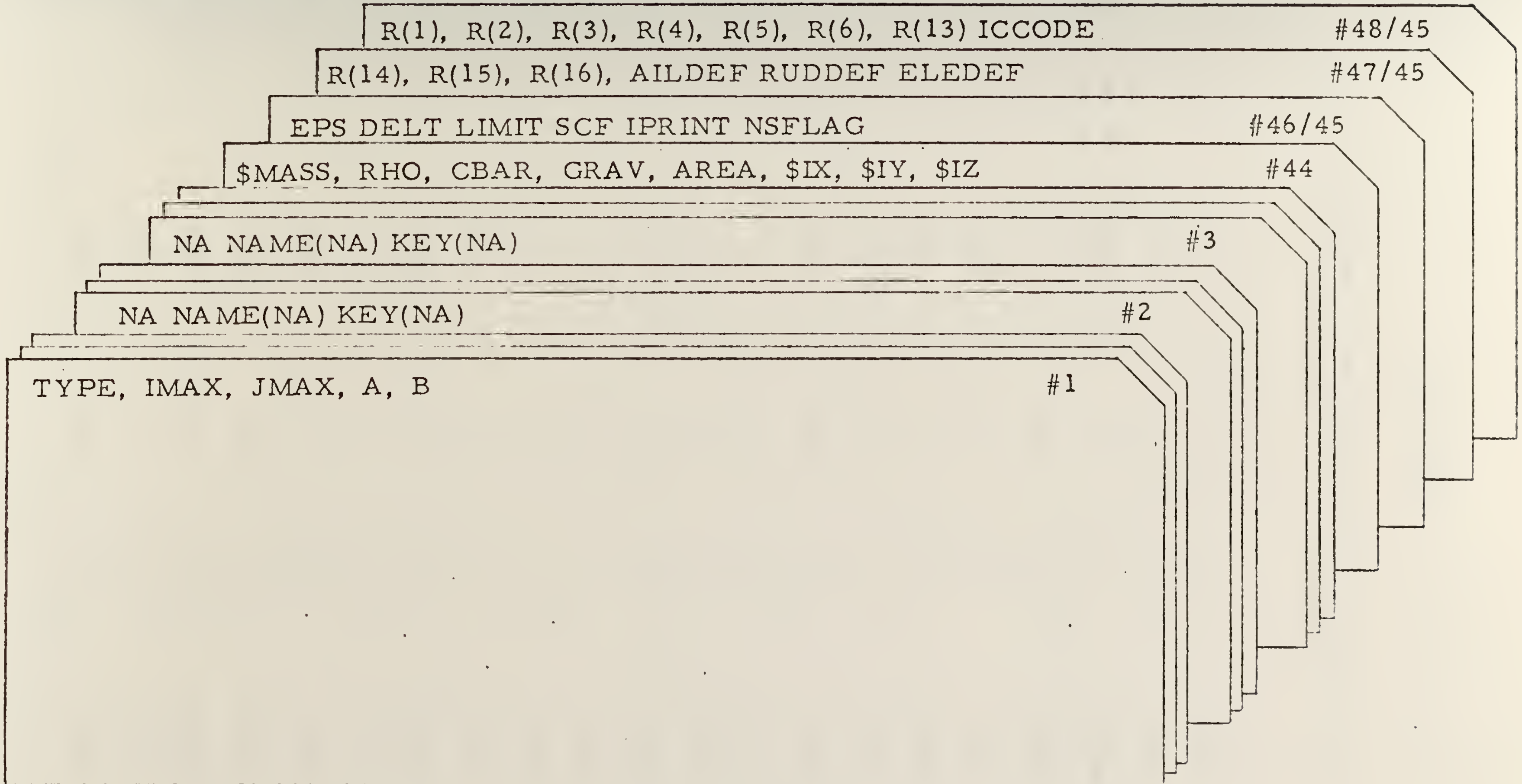


Figure B-1

Data Deck Composition for the Airplane Spin Solution Program

TABLE B-I

Force and Moment Coefficients, Stability Derivatives, and Associated 'NA' Assignments

1	C_x	CX	22	$C_{n_{\delta r}}$	CNDR
2	C_y	XY	23	C_{x_r}	CXR
3	C_z	CZ	24	$C_{x_{\delta r}}$	CXDR
4	C_l	CL	25	C_{x_p}	CXP
5	C_m	CM	26	C_{x_q}	CXQ
6	C_n	CN	27	$C_{x_{\delta a}}$	CXDA
7	$C_{x_{\delta e}}$	CZDE	28	C_{y_q}	CYQ
8	C_{y_p}	CYP	29	$C_{y_{\delta e}}$	CYDH
9	C_{y_r}	CYR	30	C_{z_p}	CZP
10	$C_{y_{\delta a}}$	CYDA	31	C_{z_q}	CZQ
11	$C_{y_{\delta r}}$	CYDR	32	C_{z_r}	CZR
12	$C_{z_{\delta e}}$	CZDE	33	$C_{z_{\delta a}}$	CZDA
13	C_{l_p}	CLP	34	$C_{z_{\delta r}}$	CZDR
14	C_{l_r}	CLR	35	C_{l_q}	CLQ
15	$C_{l_{\delta a}}$	CLDA	36	$C_{l_{\delta e}}$	CLDE
16	$C_{l_{\delta r}}$	CLDR	37	C_{m_p}	CMP
17	C_{m_q}	CMQ	38	C_{m_r}	CMR
18	$C_{m_{\delta e}}$	CMDE	39	$C_{m_{\delta a}}$	CMDA
19	C_{n_p}	CNP	40	$C_{m_{\delta r}}$	CMDR
20	C_{n_r}	CNR	41	C_{n_q}	CNQ
21	$C_{n_{\delta a}}$	CNDA	42	$C_{n_{\delta e}}$	CNDE

conditions included as data set #48 need be provided. This option is effected by assigning NSFLAG = 2 or 3 as explained in the program comments listing and Table B-II.

It is suggested that the user who wishes to test a large number of initial conditions assign LIMIT = 10, NSFLAG = 2 and then test all of the initial conditions on one job. This scheme will quickly reveal to the user which initial conditions exceed the alpha/beta limits without wasting valuable computation time. Subsequently, those initial conditions which appear to have a solution potential can be submitted with appropriate data set #45 values.

Since every iteration generates a new set of initial conditions, the program can be restarted merely by including as data set #45 the appropriate values taken from the printed output. This option allows the user to check the stability of a quasi-steady spin solution by changing the time increment (DELT), one of the solution convergence factors (SCF), the control deflections (AILDEF, RUDDEF, ELEDEF), or the atmospheric density (RHO).

C. DATA OUTPUT

All output is formated for 130 character computer paper. An option is provided by NPRINT to suppress the iterative results; otherwise, all input data is echo checked, all significant iterative computations are printed and a summary of the integration solution path is provided.

D. STORAGE REQUIREMENTS

The amount of storage required is primarily a function of the size of the CFORCM three dimensional matrix. The third index of the matrix is associated with, and is no larger than, the largest NA

TABLE B-II

NSFLAG Integer Assignment

NSFLAG	SOLUTION METHOD	EQUATION FORM	SUBSEQUENT DATA SET COMPOSITION
0	Integration	Modified	#45
1	Integration	Full	#45
2	Integration	Modified	#48
3	Integration	Full	#48
4	Gradient	Short	#45

integer (NSMAX) corresponding to vector or matrix coefficient data sets. The first and second indices are associated with IMAX and JMAX. Prior to using the program the user should dimension A, B, and CFORCM in the COMMON/WORKA/ specification statements to A(IMAX), B(JMAX), and CFORCM(IMAX, JMAX, NAMAX). The WORKA statement must be identical in the following routines: MAIN, GAWAIN, SOLVIT, EQNS, INTER, and ANGLE. Common regions are utilized for most communications between subroutines and therefore care must be exercised to ensure that the common statements are identical.

E. PROGRAM FLOWCHARTS

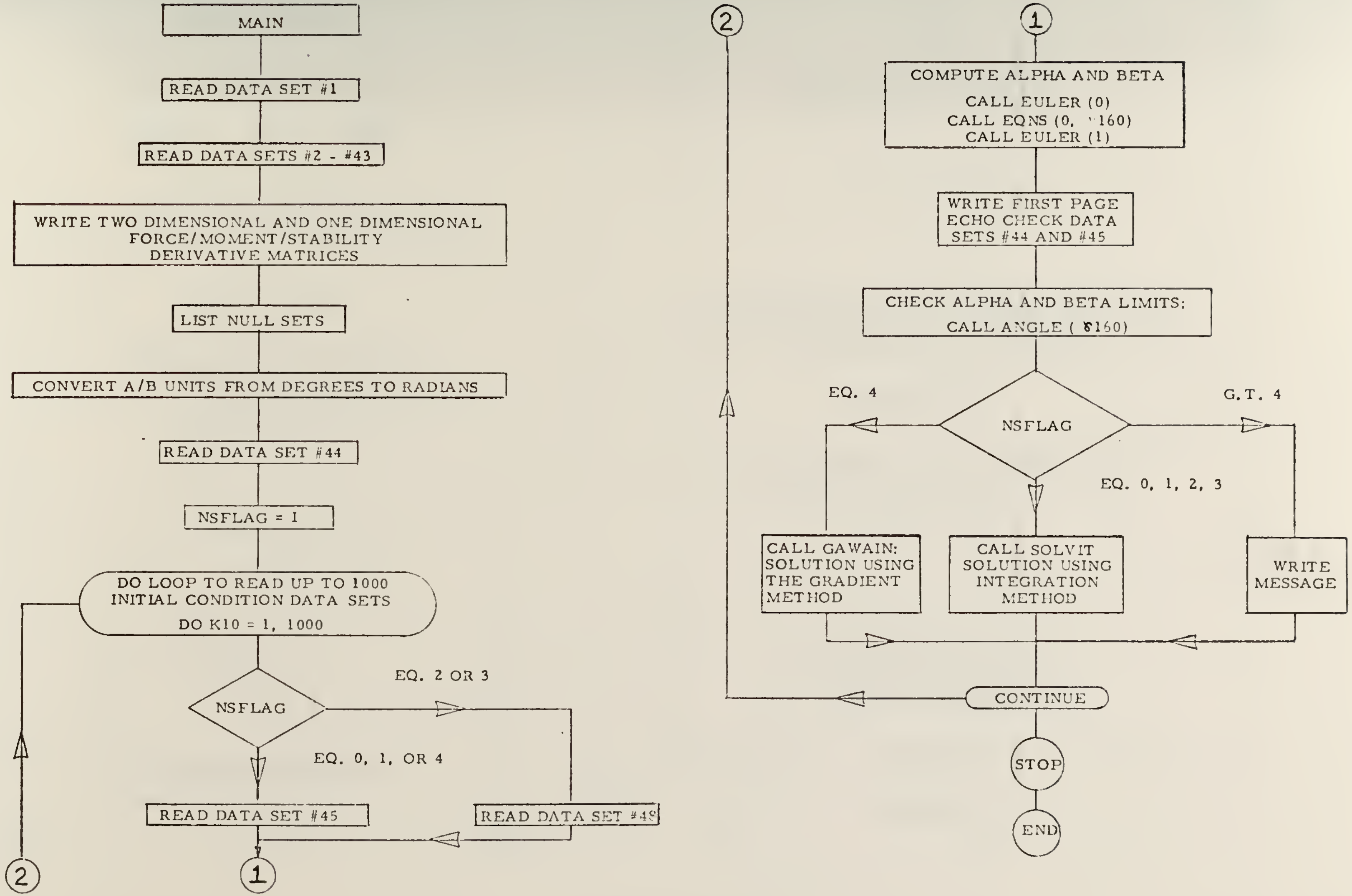
Main and subroutine flow charts are included as Figures B-2 to B-9 and are intended to be self-explanatory. The program source listing includes a sufficient number of comments to amplify the flowchart information. Subroutines SPLIN1 and SPLICO were provided by the NPS Computer Center. No effort has been made to provide reference flowcharts as both routines have been extensively tested and validated by the NPS staff.

F. PROGRAMMING CONSIDERATIONS

One should note that calls on Euler are made in pairs. The angular variables are stored in the common memory, thus if the first conversion is from equation compatible to output variables, the second call must effect the opposite conversion.

Subroutine GAWAIN requires the computation of the partial derivative of the criterion with respect to a change in each independent variable. This is accomplished by incrementing a single independent variable by

FAC. The criterion is then computed using this value. The change in the criterion function value is then divided by FAC to obtain the partial derivative.



MAIN Program Flowchart

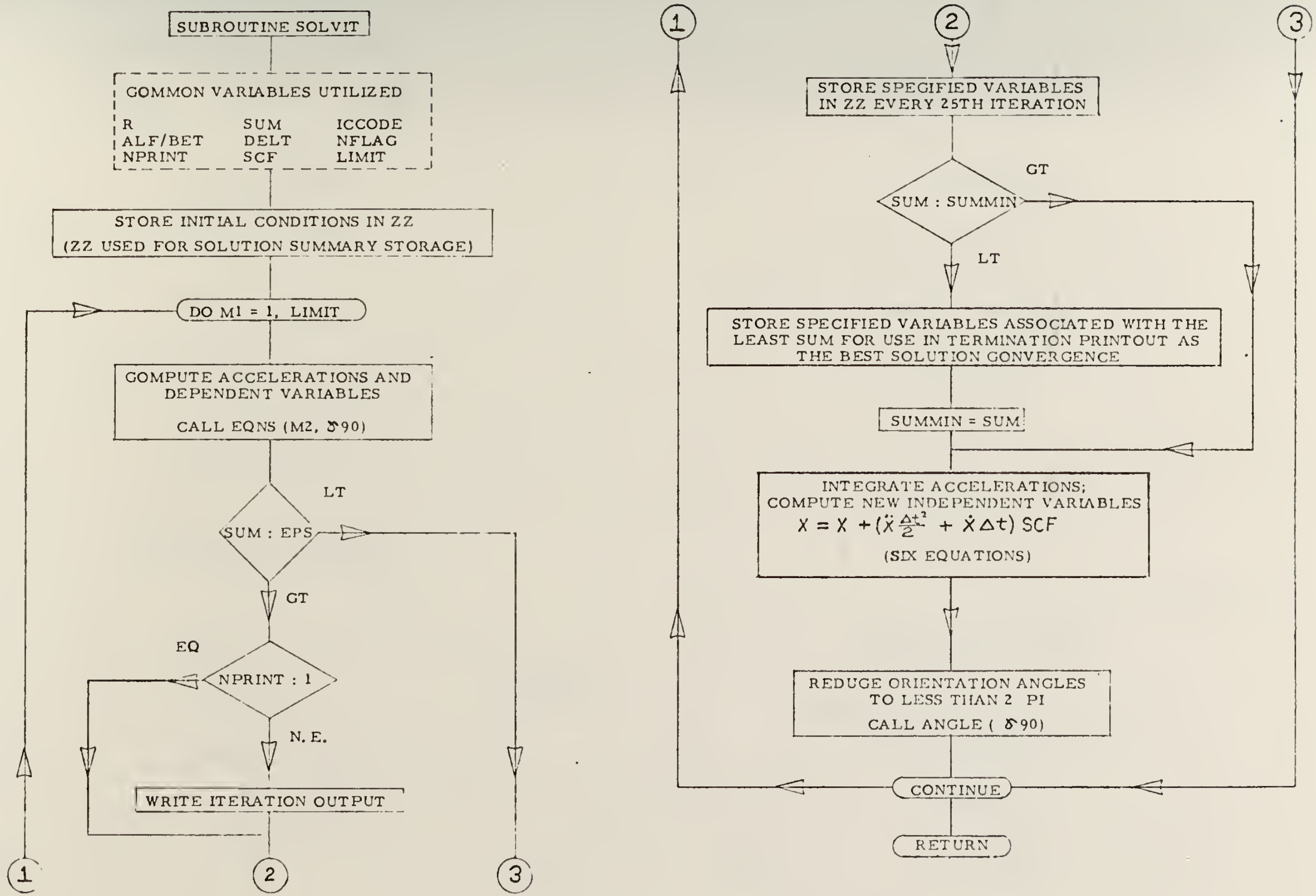


Figure B-3

SOLVIT Subroutine Flowchart

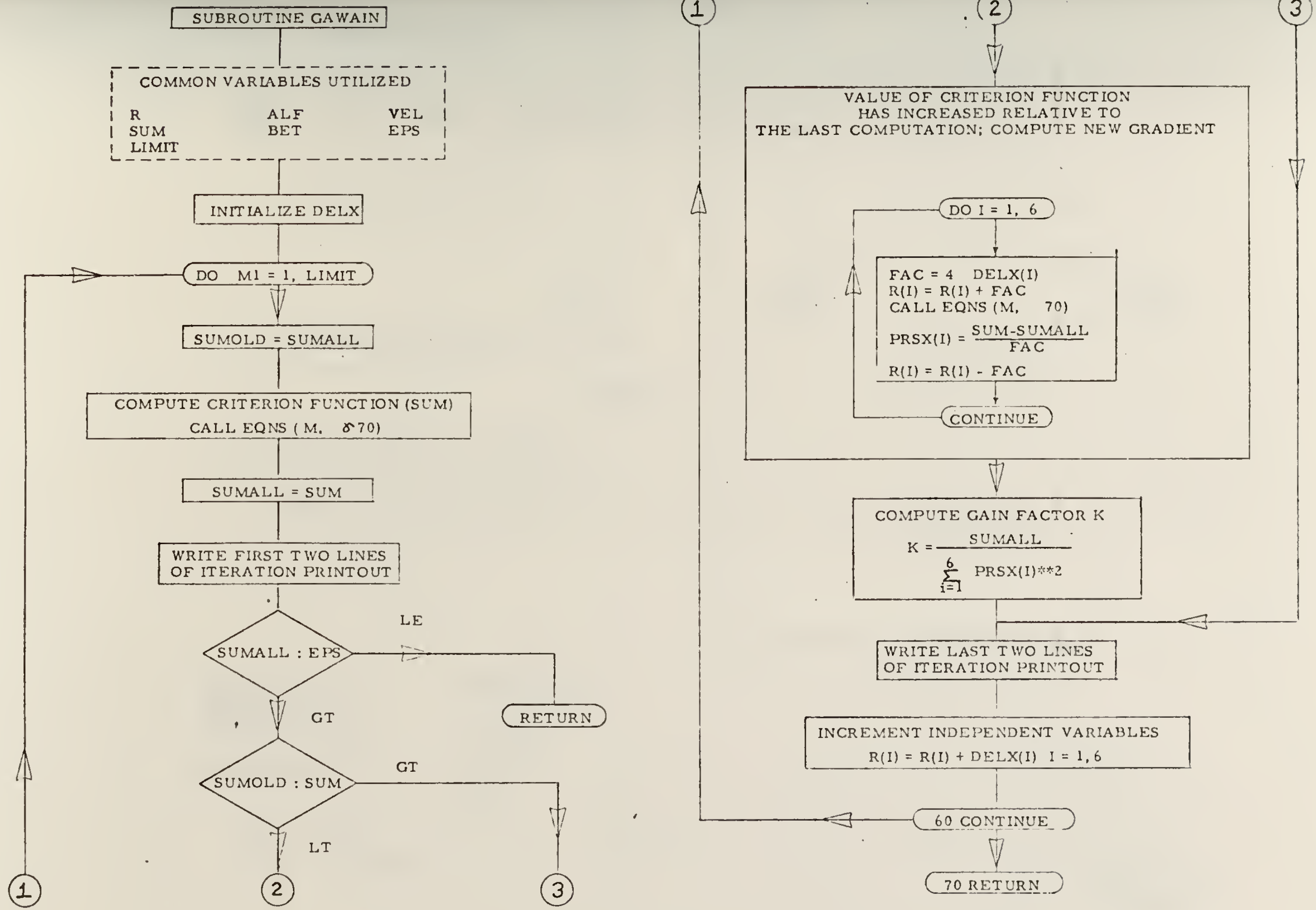


Figure B-4

GAWAIN Subroutine Flowchart

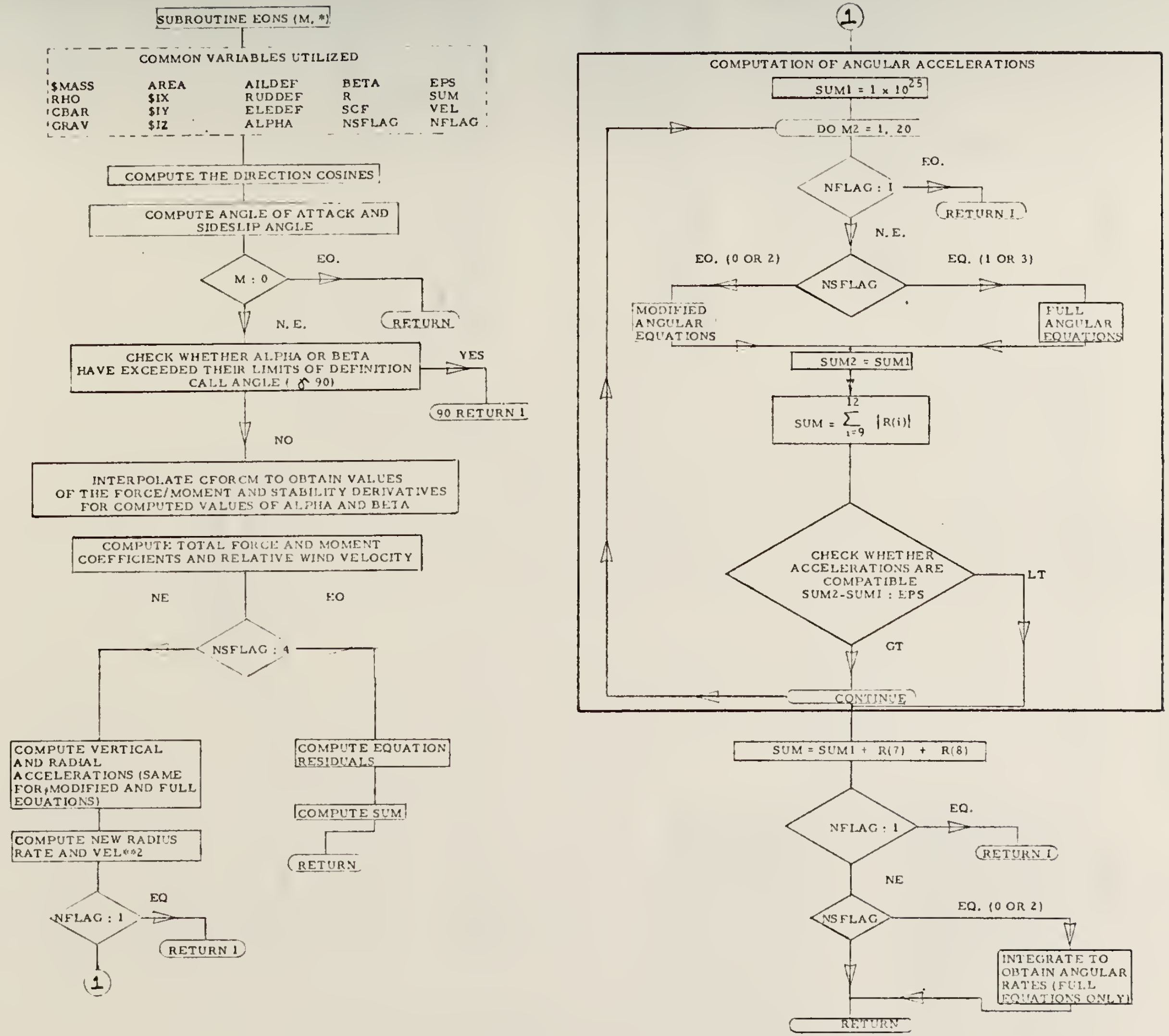


Figure B-5

EQNS Subroutine Flowchart

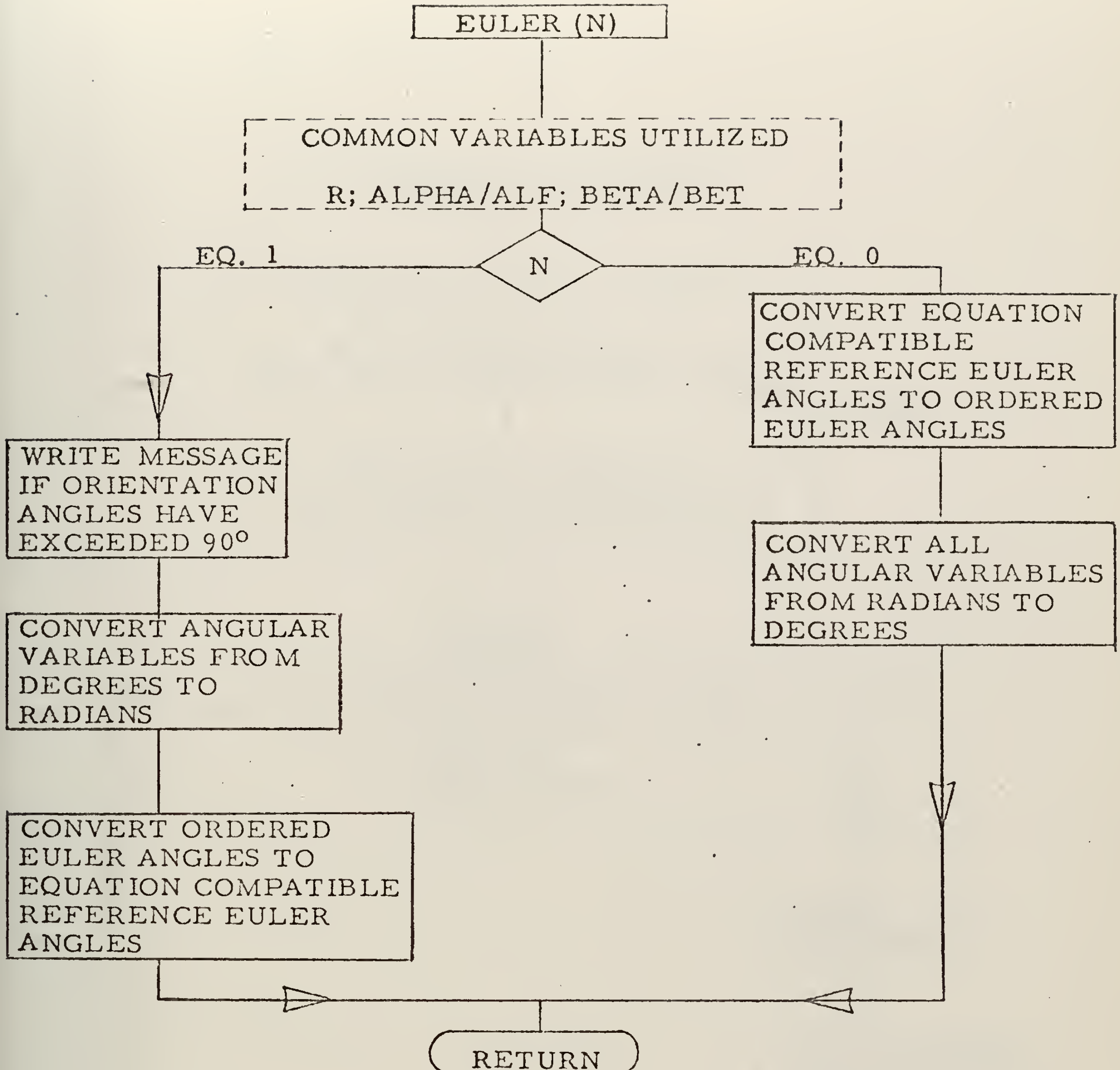


Figure B-6

EULER Subroutine Flowchart

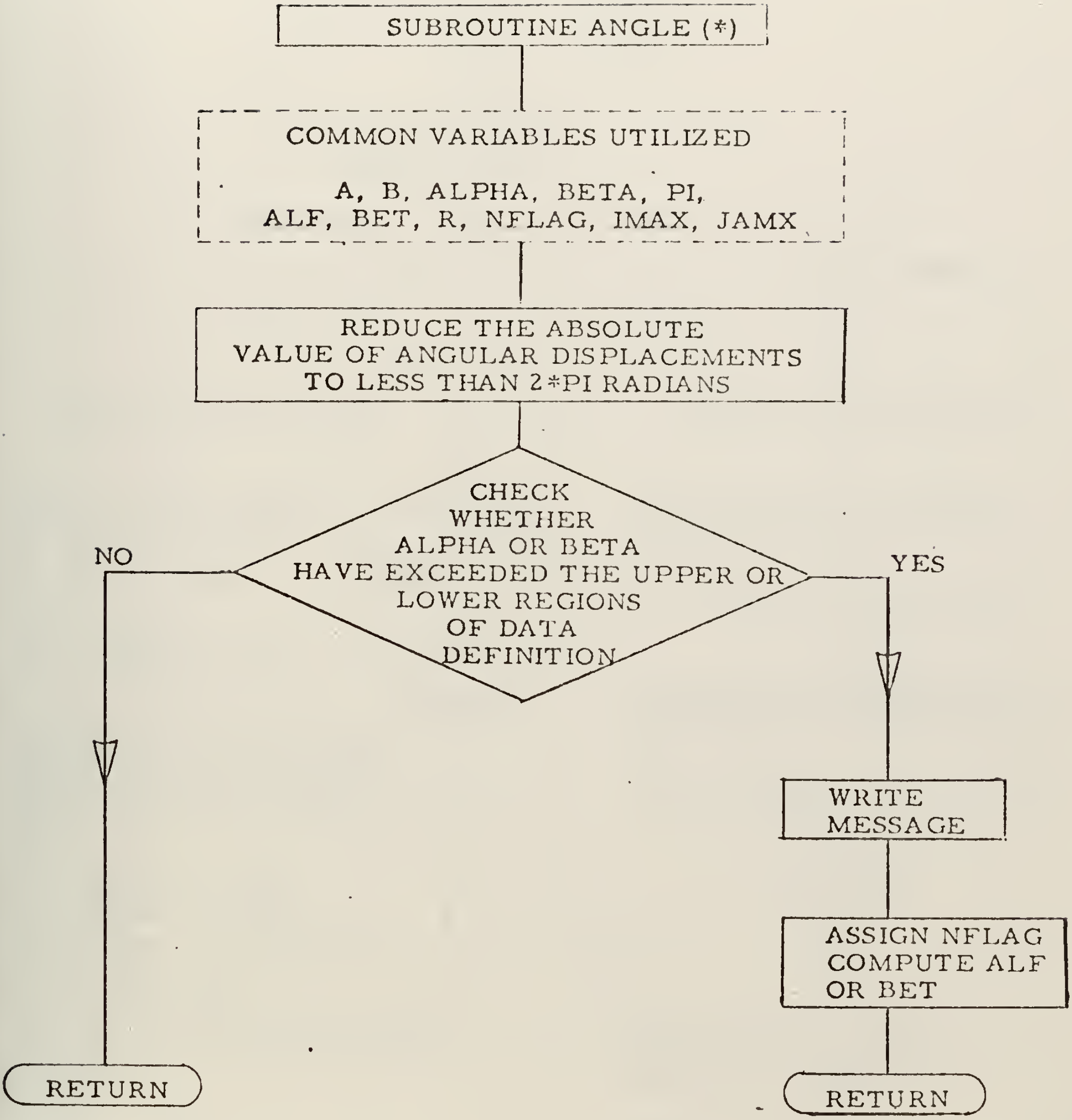


Figure B-7

ANGLE Subroutine Flowchart

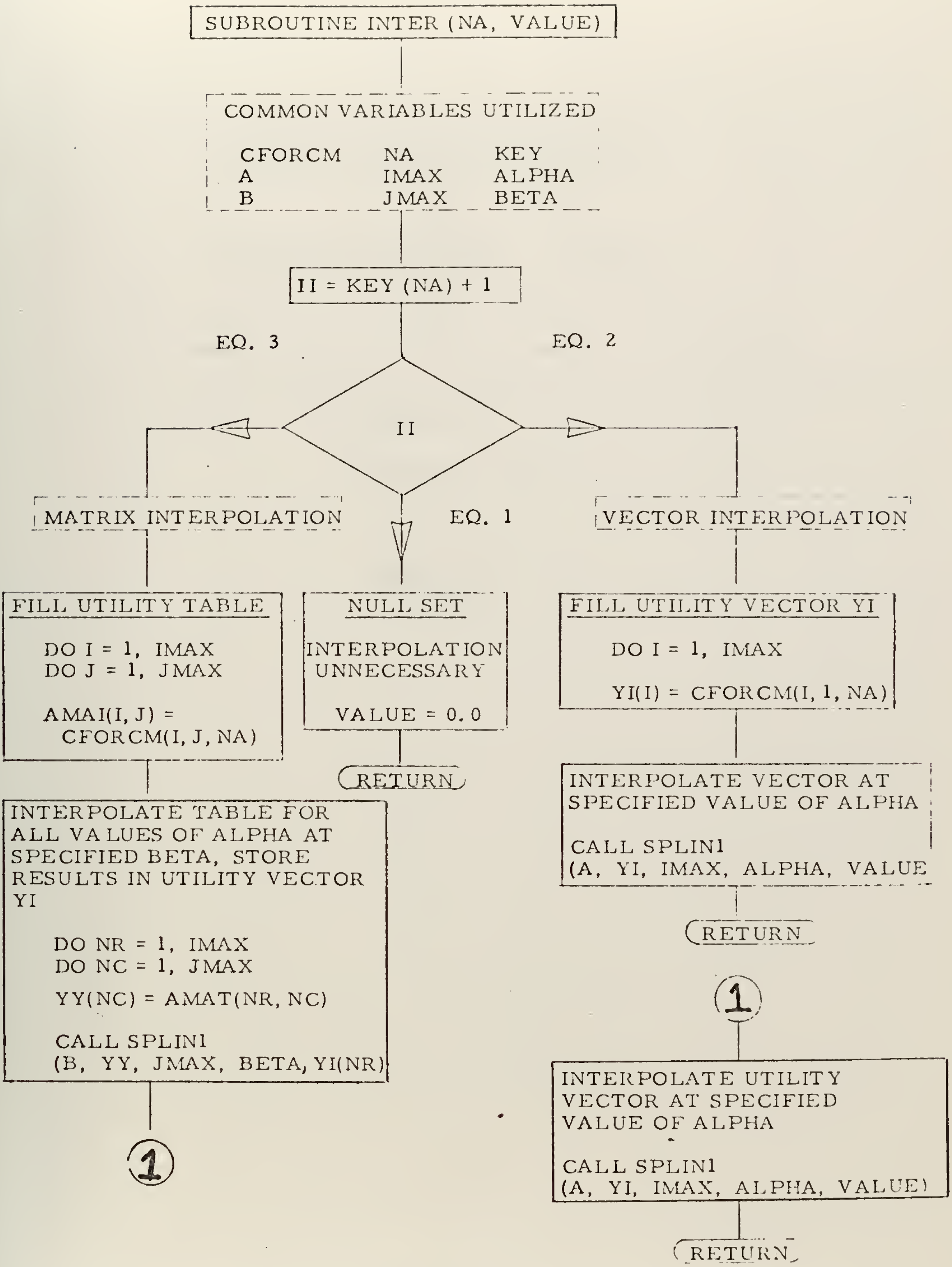


Figure B-8

INTER Subroutine Flowchart

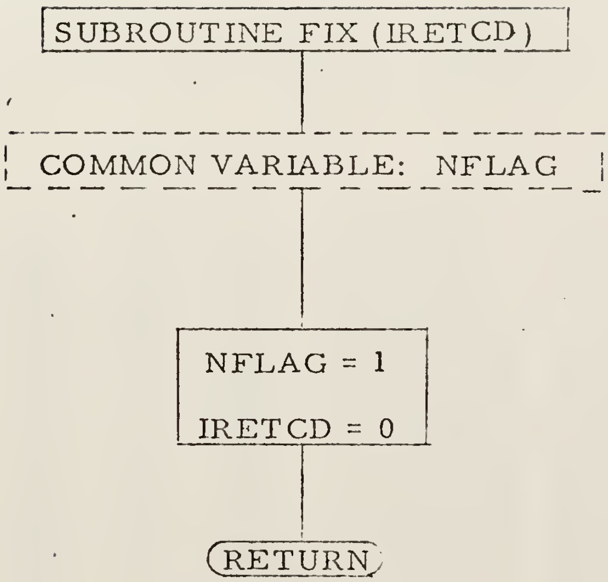


Figure B-9

FIX Subroutine Flowchart

```
C          AIRCRAFT SPIN SOLUTION PROGRAM                                      * MAI00010
C                                                                              * MAI00020
C          GENERAL INFORMATION                                                 * MAI00030
C               THIS PROGRAM SOLVES AIRPLANE EQUATIONS OF MOTION BY TWO* MAI00040
C               METHODS. THE EQUATIONS UTILIZED ARE MODELED IN A CYLIN-* MAI00050
C               DRICAL COORDINATE SYSTEM IN ORDER TO OBVIATE A STEADY  * MAI00060
C               OR QUASI STEADY STATE SOLUTION FOR A SPINNING AIRCRAFT.* MAI00070
C                                                                      * MAI00080
C          EQUATION FORMS                                              * MAI00090
C               FULL:  SIX EXTENSIVELY COUPLED, HIGHLY NON LINEAR      * MAI00100
C               DIFFERENTIAL EQUATIONS. INCLUDED ARE ALL APPLICABLE    * MAI00110
C               ZERO, FIRST, AND SECOND ORDER DERIVATIVES OF THE INDE- * MAI00120
C               PENDENT VARIABLES.                                     * MAI00130
C                                                                      * MAI00140
C               MODIFIED:  SIMILAR TO THE FULL EQUATIONS, HOWEVER THE  * MAI00150
C               DERIVATIVES OF THE ORIENTATION VARIABLES ARE EQUATED   * MAI00160
C               TO ZERO.                                               * MAI00170
C                                                                      * MAI00180
C               SHORT:  SIX ALGEBRAIC EQUATIONS WHICH ARE DERIVED FROM * MAI00190
C               THE FULL EQUATIONS BY EQUATING ALL APPLICABLE FIRST    * MAI00200
C               AND SECOND ORDER DERIVATES TO ZERO.                    * MAI00210
C                                                                      * MAI00220
C          SOLUTION METHODS                                            * MAI00230
C               INTEGRATION METHOD:  ITERATIVE SOLUTION WHEREBY THE    * MAI00240
C               INDEPENDENT VARIABLE ACCELERATIONS ARE COMPUTED AS A   * MAI00250
C               FUNCTION OF THE INPUTTED INITIAL CONDITIONS. NEW INI-  * MAI00260
C               TIAL CONDITIONS ARE THEN COMPUTED BY INTEGRATING THE   * MAI00270
C               ACCELERATIONS. A SOLUTION IS OBTAINED WHEN THE ACCEL-  * MAI00280
C               ERATIONS REACH A MINIMUM VALUE. THIS METHOD IS APPLI-  * MAI00290
C               CABLE ONLY TO THE FULL AND MODIFIED EQUATIONS.         * MAI00300
C                                                                      * MAI00310
C               GRADIENT METHOD:  ITERATIVE SOLUTION WHEREBY A CRITE-  * MAI00320
C               RION FUNCTION IS COMPUTED FROM INPUTTED INITIAL CONDI- * MAI00330
C               TIONS; THE GRADIENT TO THE FUNCTION IS USED TO GENERATE* MAI00340
C               NEW INITIAL CONDITIONS. A STEADY STATE SOLUTION IS     * MAI00350
C               ACHIEVED WHEN THE CRITERION FUNCTION EQUALS ZERO. THIS * MAI00360
C               METHOD IS APPLICABLE TO THE SHORT EQUATIONS ONLY.      * MAI00370
C                                                                      * MAI00380
C          DESCRIPTION OF PARAMETERS (AIRCRAFT/ENVIRONMENTAL CONSTANTS)* MAI00390
C               $MASS:         MASS OF AIRCRAFT, SLUGS                 * MAI00400
C               CBAR:          MEAN AERODYNAMIC CHORD, FEET            * MAI00410
C               AREA:          WING AREA, FEET**2                      * MAI00420
C               $IX,$IY,$IZ:   PRINCIPAL MOMENTS OF INERTIA,           * MAI00430
C                              SLUG-FEET**2                            * MAI00440
C               TYPE:          AIRCRAFT TYPE, I.E., F-4, F111          * MAI00450
```

```
C     NA:           INTEGER CORRESPONDING TO A PARTICULAR     * MAI00460
C                   STABILITY DERIVATIVE. (REFER TO THE FIRST* MAI00470
C                   REFERENCE FOR THE INTEGER ASSIGNMENT)     * MAI00480
C     KEY(42):      VECTOR OF INTEGERS WHICH INDICATE FORM OF* MAI00490
C                   STABILITY DERIVATIVE                      * MAI00500
C                   KEY = 0:  NULL SET                        * MAI00510
C                   KEY = 1:  1-D VECTOR THAT IS A FUNCTION   * MAI00520
C                   ONLY OF ALPHA                             * MAI00530
C                   KEY = 2:  2-D MATRIX THAT IS A FUNCTION   * MAI00540
C                   OF BOTH ALPHA AND BETA                    * MAI00550
C     NAME(42):     VECTOR OF LITERAL NAMES OF THE STABILITY * MAI00560
C                   DERIVATIVES                               * MAI00570
C     RHO:          AIR DENSITY, SLUG/FEET**3                 * MAI00580
C     GRAV:         GRAVITATIONAL CONSTANTS, FEET/SECOND**2   * MAI00590
C     CFORCM:       THREE DIMENSIONAL DATA BANK CONTAINING    * MAI00600
C                   THE STABILITY DERIVATIVE MATRICES.        * MAI00610
C                   DIMENSIONED TO AT LEAST (IMAX,JMAX,NAMAX)* MAI00620
C                   WHERE NAMAX IS EQUAL TO THE LARGEST INTE-* MAI00630
C                   GER CONTAINED IN KEY                      * MAI00640
C     A(IMAX):      VECTOR OF THE VALUES OF AIRCRAFT ANGLE OF* MAI00650
C                   ATTACK FOR WHICH THERE IS A ONE-FOR-ONE   * MAI00660
C                   CORRESPONDENCE TO THE ORDINATE VALUES     * MAI00670
C                   OF THE STABILITY DERIVATIVE MATRICES,     * MAI00680
C                   DEGREES                                   * MAI00690
C     B(JMAX):      VECTOR OF THE VALUES OF AIRCRAFT SIDESLIP* MAI00700
C                   ANGLE FOR WHICH THERE IS A ONE-FOR-ONE    * MAI00710
C                   CORRESPONDENCE TO THE ABSCISSA VALUES     * MAI00720
C                   OF THE STABILITY DERIVATIVE MATRICES,     * MAI00730
C                   DEGREES                                   * MAI00740
C     IMAX/JMAX:    INTEGER EQUAL TO THE NUMBER OF VALUES     * MAI00750
C                   IN THE A/B VECTOR                         * MAI00760
C     ALPHA/ALF:    COMPUTED ANGLE OF ATTACK, RADIANS/DEGREES* MAI00770
C     BETA/BET:     COMPUTED SIDESLIP ANGLE, RADIANS/DEGREES * MAI00780
C                                                             * MAI00790
C   DESCRIPTION OF PARAMETERS (SOLUTION VARIABLES)            * MAI00800
C     DELT:         INTEGRATION TIME INCREMENT, SECONDS       * MAI00810
C     SCF(6):       SOLUTION CONVERGENCE FACTORS              * MAI00820
C     LIMIT:        ITERATION DO LOOP LIMIT                   * MAI00830
C     NFLAG:        INTEGER ASSOCIATED WITH THE EXTENDED      * MAI00840
C                   ERROR HANDLING FACILITY.(REFER TO SUBROU-* MAI00850
C                   TINE 'FIX') ALSO USED TO COMMUNICATE      * MAI00860
C                   ABNORMAL TERMINATION INFORMATION.           MAI00870
C     NSFLAG:       INTEGER WHICH DETERMINES THE SOLUTION     * MAI00880
C                   METHOD AND FORM OF EQUATIONS TO BE        * MAI00890
C                   UTILIZED                                  * MAI00900
C     NPRINT:       INTEGER WHICH IF EQUAL TO ONE WILL SUP-   * MAI00910
C                   PRESS THE ITERATION OUTPUT BUT WILL       * MAI00920
C                   PROVIDE A SUMMARY OF THE SOLUTION         * MAI00930
```

```
C       ICCODE:         SIX DIGIT INTEGER SERIAL NUMBER OF INPUT * MAI00940
C                       INITIAL CONDITONS                        * MAI00950
C       SUM:            SUM OF THE ABSOLUTE VALUES OF THE INDE-  * MAI00960
C                       PENDENT VARIABLE ACCELERATIONS           * MAI00970
C       EPS:            REAL VALUE USED TO DETERMINE CONVERGENCE * MAI00980
C       AILDEF:         AIRCRAFT AILERON DEFLECTION, DEGREES     * MAI00990
C                       RIGHT AILERON TRAILING EDGE DOWN IS POS- * MAI01000
C                       TIVE                                     * MAI01010
C       RUDDEF:         AIRCRAFT RUDDER DEFLECTION, DEGREES      * MAI01020
C                       TRAILING EDGE LEFT IS POSITIVE           * MAI01030
C       ELEDEF:         AIRCRAFT ELEVATOR OR STABILATOR DEFLEC-  * MAI01040
C                       TION, DEGREES, TRAILING EDGE DOWN IS     * MAI01050
C                       POSITIVE.                                * MAI01060
C                                                                * MAI01070
C   DESCRIPTION OF PARAMETERS (INDEPENDENT VARIABLES)            * MAI01080
C       R:              VECTOR OF THE INDEPENDENT VARIABLES AND  * MAI01090
C                       ASSOCIATED DERIVATIVES AS DESCRIBED BELOW* MAI01100
C                                                                * MAI01110
C       R(1):           ALTITUDE RATE, FEET/SEC, NEGATIVE DIREC- * MAI01120
C                       TION IS DOWNWARDS                        * MAI01130
C       R(2):           RADIUS, (FEET)                           * MAI01140
C       R(3):           SPINRATE, DEGREES/SECOND                 * MAI01150
C       R(4):           THETA, PITCH EULER ANGLE, DEGREES        * MAI01160
C       R(5):           PSI, ROLL EULER ANGLE, DEGREES           * MAI01170
C       R(6):           PHI, YAW EULER ANGLE, DEGREES            * MAI01180
C       R(7):           VERTICAL ACCELERATION, FEET/SECOND**2    * MAI01190
C       R(8):           RADIUS ACCELERATION, FEET/SECOND**2      * MAI01200
C       R(9):           ANGULAR ACCELERATION, RADIANS/SECOND**2  * MAI01210
C       R(10):          PITCH ACCELERATION, RADIANS/SECOND**2    * MAI01220
C       R(11):          ROLL ACCELERATION, RADIANS/SECOND**2     * MAI01230
C       R(12):          YAW ACCELERATION, RADIANS/SECOND**2      * MAI01240
C       R(13):          RADIUS RATE, FEET/SECOND                 * MAI01250
C       R(14):          PITCH RATE, RADIANS/SECOND               * MAI01260
C       R(15):          ROLL RATE, RADIANS/SECOND                * MAI01270
C       R(16):          YAW RATE, RADIANS/SECOND                 * MAI01280
C                                                                * MAI01290
C       NOTE:           ORIENTATION ORDER IS  PITCH, ROLL AND    * MAI01300
C                       THEN YAW                                 * MAI01310
C                                                                * MAI01320
C   USAGE                                                        * MAI01330
C       THE PROGRAM READS THE STABILITY DERIVATIVE MATRICES      * MAI01340
C       AND RELATED AIRCRAFT CONSTANTS AND THEN COMMENCES TO     * MAI01350
C       ITERATIVELY SEEK A SOLUTION BASED UPON THE FIRST INI-    * MAI01360
C       TIAL CONDITION DATA SET THAT IS READ.  UPON TERMINATION* MAI01370
C       OF A PARTICULAR SOLUTION, THE PROGRAM THEN READS SUBSE-* MAI01380
C       QUENT INITIAL CONDITION DATA SETS; THE FORMAT OF WHICH * MAI01390
C       IS EXPLAINED BELOW UNDER 'DATA DECK COMPOSITION'         * MAI01400
C                                                                * MAI01410
```

```
C     SUBROUTINES REQUIRED/PURPOSE                                   * MAI01420
C          SOLVIT:        INTEGRATION SOLUTION                       * MAI01430
C          GAWAIN:        GRADIENT SOLUTION                          * MAI01440
C          EQNS:          EQUATIONS (THREE FORMS)                    * MAI01450
C          ANGLE:         CHECKS ALPHA AND BETA LIMITS               * MAI01460
C          EULER:         CONVERTS EULER ANGLES                      * MAI01470
C          INTER:         INTERPOLATION CONTROL                      * MAI01480
C          SPLIN:         CUBIC INTERPOLATION                        * MAI01490
C          ERRSET         OVERRIDES FORTRAN ERROR HANDLING FACILITY* MAI01500
C          FIX            FACILITATES EXECUTION TERMINATION          * MAI01510
C                                                                    * MAI01520
C     SOLUTION METHOD AND EQUATION FORM                              * MAI01530
C          NSFLAG = 0 OR 2:  MODIFIED EQUATIONS/INTEGRATION METHOD* MAI01540
C          NSFLAG = 1 OR 3:  FULL EQUATIONS/INTEGRATION METHOD       * MAI01550
C          NSFLAG = 4:       SHORT EQUATIONS/GRADIENT METHOD         * MAI01560
C                                                                    * MAI01570
C     PRINTED OUTPUT                                                 * MAI01580
C          THE AMOUNT OF PRINTED OUTPUT IS DETERMINED BY NPRINT.     * MAI01590
C          THE DEFAULT VALUE OF (0) PROVIDES AN ECHO CHECK OF ALL    * MAI01600
C          INITIAL CONDITIONS AND SOLUTION PARAMETERS, A PRINTOUT    * MAI01610
C          OF EACH SOLUTION ITERATION AND, AFTER TERMINATION, A      * MAI01620
C          SUMMARY OF THE SOLUTION PATH IS PROVIDED                  * MAI01630
C                                                                    * MAI01640
C     JOB CONTROL LANGUAGE (EXECUTION, USING SOURCE DECK)            * MAI01650
C          1. //JOB                                                  * MAI01660
C          2. // EXEC FORTCLG,REGION.GO=135K  (FORTRAN IV G)         * MAI01670
C          OR // EXEC FORTHCLG,PARM.FORT='OPT=2',REGION.GO=135K      * MAI01680
C             (FORTRN IV H)                                          * MAI01690
C          3. SOURCE DECK                                            * MAI01700
C          4. /*                                                     * MAI01710
C          5. //GO.FT06F001 DD SYSOUT=A,SPACE=(CYL(3,2))             * MAI01720
C          6. //GO.SYSIN DD *                                        * MAI01730
C          7. DATA DECK                                              * MAI01740
C          8. /*                                                     * MAI01750
C                                                                    * MAI01760
C     REFERENCES                                                     * MAI01770
C          CHAMPOUX, R.L. 'THE NUMERICAL SOLUTION AND ANALYSIS OF * MAI01780
C          AIRPLANE SPIN EQUATIONS MODELED IN AN INERTIAL COORDI- * MAI01790
C          NATE SYSTEM', MASTERS THESIS, UNITED STATES NAVAL POST-* MAI01800
C          GRADUATE SCHOOL, MONTEREY, CALIFORNIA, DECEMBER 1972      * MAI01810
C                                                                    * MAI01820
C          BUEHLER, R.R. 'THE DERIVATION, SOLUTION, AND ANALYSIS     * MAI01830
C          OF AIRPLANE SPIN EQUATIONS MODELED IN AN INERTIAL COOR-* MAI01840
C          DINATE SYSTEM', MASTERS THESIS, UNITED STATES NAVAL       * MAI01850
C          POST GRADUATE SCHOOL, MONTEREY, CALIFORNIA, MARCH 1972 * MAI01860
C                                                                    * MAI01870
C                                                                    * MAI01880
C                                                                    * MAI01890
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C                     DATA DECK COMPOSITION                          * MAI01900
C********************************************************************* MAI01910
C DATA SET *              VARIABLES                   *       FORMAT   * MAI01920
C********************************************************************* MAI01930
C    1     * TYPE,IMAX,JMAX,(A(I),I=1,IMAX),        * (A4,2I3,7F10.2,    * MAI01940
C          *     (B(J),J=1,JMAX)                    *    (8F10.2))       * MAI01950
C********************************************************************* MAI01960
C          * NA, NAME(NA), KEY(NA)  THE REMAINDER* (I10,10X,A4,10X,I6)* MAI01970
C          *   EACH DATA SET IS DETERMINED BY KEY*                   * MAI01980
C          *********************************************************** MAI01990
C          *           IF(KEY.EQ.2)              *                   * MAI02000
C          * ((CFORCM(I,J,NA),J=1,JMAX),I=1,IMAX)*    (8F10.6)       * MAI02010
C    2-43  *********************************************************** MAI02020
C(42 SETS) *           IF(KEY.EQ.1)              *                   * MAI02030
C          *  (CFORCM(I,1,NA),I=1,IMAX)          *    (8F10.6)       * MAI02040
C          *********************************************************** MAI02050
C          *  IF(KEY.EQ.0) THE NA STABILITY      *                   * MAI02060
C          * DERIVATIVE MATRIX IS NULL AND NO    * (NOT APPLICABLE)  * MAI02070
C          * DATA CARDS WILL BE READ             *                   * MAI02080
C********************************************************************* MAI02090
C   44     * $MASS,RHO, CBAR, GRAV, AREA, $IX    *    (8F10.6)       * MAI02100
C          *  $IY, $IZ                           *                   * MAI02110
C********************************************************************* MAI02120
C     * 46 * EPS, DELT, LIMIT, (SCF(I),I=1,6),   * (2F10.6,I10,6F5.2, * MAI02130
C     *    *  NPRINT, NSFLAG                     *    2I10)          * MAI02140
C     **************************************************************** MAI02150
C     * 47 * R(14), R(15), R(16), AILDEF, RUDDEF,*    (8F10.6)       * MAI02160
C 45 *     *  ELEDEF                             *                   * MAI02170
C     **************************************************************** MAI02180
C     * 48 * (R(I),I=1,6), R(13), ICCODE         *    (7F10.6,I10)   * MAI02190
C********************************************************************* MAI02200
C                                                                    * MAI02210
C               NOTE:    THE FIRST SET OF INITIAL CONDITIONS MUST BE * MAI02220
C                        PROVIDED AS DATA SET #45.  THE COMPOSITION  * MAI02230
C                        OF THE SUBSEQUENT SETS OF INITIAL CONDITIONS * MAI02240
C                        IS DETERMINED BY NSFLAG                     * MAI02250
C                                                                    * MAI02260
C                        IF NSFLAG= 0,1,OR 4, THE USER MUST PROVIDE  * MAI02270
C                        DATA SET #45 AS THE NEXT SET OF INITIAL     * MAI02280
C                        CONDITIONS                                  * MAI02290
C                                                                    * MAI02300
C                        IF NSFLAG = 2 OR 3, THE VARIABLES IN DATA   * MAI02310
C                        SETS #46 AND #47 WILL REMAIN CONSTANT FOR ALL* MAI02320
C                        SUBSEQUENT COMPUTATIONS. THE USER MAY ONLY  * MAI02330
C                        PROVIDE SUBSEQUENT INITIAL CONDITIONS AS DATA* MAI02340
C                        SET #48 AND ALL COMPUTATIONS WILL BE MADE   * MAI02350
C                        USING THE INTEGRATION METHOD                * MAI02360
C                                                                    * MAI02370
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      IMPLICIT REAL *8 (A-H,O-Z,$)                                        MAI02380
      COMMON NFLAG,NPRINT,ICCODE                                          MAI02390
      COMMON/WORKA/$MASS,RHO,CBAR,GRAV,AREA,$IX,$IY,$IZ,PI,IMAX,JMAX,     MAI02400
     1A(22),B(9),CFORCM(22,9,22),KEY(42),AILDEF,RUDDEF,ELEDEF             MAI02410
      COMMON/WORKB/ALPHA,ALF,BETA,BET,R(16),DELT,SCF(6),LIMIT,NSFLAG,     MAI02420
     1EPS,SUM ,VEL                                                        MAI02430
      REAL NAME(42),TYPE                                                  MAI02440
      INTEGER P(9),A1,A2                                                  MAI02450
      DIMENSION N1(42),N2(42),NO(42)                                      MAI02460
      CALL ERRSET(207,100,-1,1,FIX,209)                                   MAI02470
      NOCNTR=0                                                            MAI02480
      N1CNTR=0                                                            MAI02490
      N2CNTR=0                                                            MAI02500
C                                                                        *MAI02510
C          READ DATA SET #1                                              *MAI02520
C                                                                        *MAI02530
      READ(5,1000) TYPE,IMAX,JMAX,(A(II),II=1,IMAX),(B(J),J=1,JMAX)       MAI02540
C                                                                        *MAI02550
C          READ DATA SETS #2 - #43                                       *MAI02560
C                                                                        *MAI02570
      DO   30 K1=1,42                                                     MAI02580
      READ (5,1001)NA,NAME(NA),KEY(NA)                                    MAI02590
      IF(KEY(NA).EQ.1)READ(5,1002)(CFORCM(I,1,NA),I=1,IMAX)               MAI02600
      IF(KEY(NA).EQ.2)READ(5,1002)((CFORCM(I,J,NA),J=1,JMAX),I=1,IMAX)    MAI02610
      IF(KEY(NA).EQ.0)GO TO 20                                            MAI02620
      IF(KEY(NA).EQ.2) GO TO   10                                         MAI02630
C                                                                        *MAI02640
C          COUNT THE NUMBER OF STABILITY DERIVATIVE MATRICES WHICH       *MAI02650
C          HAVE KEY=0/1/2; PLACE NA VALUES IN THE APPROPRIATE QUEUES     *MAI02660
C          (NO, N1, N2)                                                  *MAI02670
C                                                                        *MAI02680
      N1CNTR=N1CNTR+1                                                     MAI02690
      N1(N1CNTR)=NA                                                       MAI02700
      GO TO   30                                                          MAI02710
   10 N2CNTR=N2CNTR+1                                                     MAI02720
      N2(N2CNTR)=NA                                                       MAI02730
      GO TO   30                                                          MAI02740
   20 NOCNTR=NOCNTR+1                                                     MAI02750
      NO(NOCNTR)=NA                                                       MAI02760
   30 CONTINUE                                                            MAI02770
      IF(NPRINT.EQ.1) GO TO 120                                           MAI02780
      NOMAX=NOCNTR                                                        MAI02790
      N1MAX=N1CNTR                                                        MAI02800
      N2MAX=N2CNTR                                                        MAI02810
C                                                                        *MAI02820
C          WRITE THE TWO DIMENSIONAL MATRICES  (N2 QUEUE)                *MAI02830
C                                                                        *MAI02840
      DO  60 K2=1,N2MAX                                                   MAI02850
```

```
      DO 40 J=1,JMAX
   40 P(J)=IDINT(B(J))
      I1=N2(K2)
      WRITE(6,2000) TYPE.NAME(I1),(P(J),J=1,JMAX)
      DO   50 K3=1,IMAX
      A1=IDINT(A(K3))
   50 WRITE(6,2010 )A1,(CFORCM(K3,J,I1),J=1,JMAX)
   60 CONTINUE
C
C         WRITE THE VECTORS (N1 QUEUE)
C
      M1=1
      M2=10
      DO   80 K4=1,3
      IF(N1MAX.LE.10)M2=N1MAX
      WRITE(6,2020) TYPE,(NAME(N1(I)),I=M1,M2)
      DO 65 K6=1,IMAX
      A2=IDINT(A(K6))
   65 WRITE(6,2010)A2,(CFORCM(K6,1,N1(I)),I=M1,M2)
      M1=M1+10
      IF(M2.EQ.N1MAX)GO TO 90
      M2=M2+10
      IF(N1MAX-M2)  70,  80,  80
   70 M2=N1MAX
   80 CONTINUE
C
C         LIST THE NULL SETS (NO QUEUE)
C
   90 WRITE(6,2030)
      M1=1
      M2=7
      DO 110 K7=1,4
      IF(NOMAX.LE.7 )M2=NOMAX
      WRITE(6,2040)(NAME(NO(I)),I=M1,M2)
      M1=M1+7
      IF(M2.EQ.NOMAX)GO TO 120
      M2=M2+7
      IF(NOMAX-M2)  100,110,110
  100 M2=NOMAX
  110 CONTINUE
  120 PI=3.14159265358979
C
C         CONVERT A/B VECTORS TO RADIANS
C
      DO 130 I=1,IMAX
  130 A(I)=A(I)*PI/180.0
      DO  140 J=1,JMAX
  140 B(J)=B(J)*PI/180.0
```

```
C                                                                         * MAI03340
C         READ DATA SET #44                                               * MAI03350
C                                                                         * MAI03360
      READ(5,1002)$MASS,RHO,CBAR,GRAV,AREA,$IX,$IY,$IZ                      MAI03370
C                                                                         * MAI03380
C         NOW SET NSFLAG TO 1 IN ORDER TO ENSURE THAT DATA SET #45        * MAI03390
C         IS READ ON THE FIRST PASS                                       * MAI03400
C                                                                         * MAI03410
      NSFLAG=1                                                              MAI03420
C                                                                         * MAI03430
C         DO LOOP WHICH PROVIDES OPPORTUNITY FOR THE USER TO INITIATE     * MAI03440
C         UP TO 1000 ITERATIVE SOLUTIONS                                  * MAI03450
C                                                                         * MAI03460
      DO  160 K10=1,1000                                                    MAI03470
      IF((NSFLAG.EQ.2).OR.(NSFLAG.EQ.3))GO TO 155                           MAI03480
C                                                                         * MAI03500
C         IF NSFLAG = 2 OR 3,  DATA SETS #46 AND #47 WILL REMAIN          * MAI03510
C         CONSTANT AND ONLY DATA SET #48 WILL BE READ                     * MAI03520
C                                                                         * MAI03530
      READ(5,1003,END=170) EPS,DELT,LIMIT,SCF,NPRINT,NSFLAG                 MAI03490
      READ(5,1002)R(14),R(15),R(16),AILDEF,RUDDEF,ELEDEF                    MAI03540
C                                                                         * MAI03550
C         SAVE THE INITIAL ORIENTATION RATES IF NSFLAG = 3                * MAI03560
C                                                                         * MAI03570
      STHDOT=R(14)                                                          MAI03580
      SPSDOT=R(15)                                                          MAI03590
      SPHDOT=R(16)                                                          MAI03600
  155 READ(5,1004,END= 170)(R(I),I=1,6),R(13),ICCODE                        MAI03610
C                                                                         * MAI03620
C         RESET THE INITIAL ORIENTATION RATES                             * MAI03630
C                                                                         * MAI03640
      R(14)=STHDOT                                                          MAI03650
      R(15)=SPSDOT                                                          MAI03660
      R(16)=SPHDOT                                                          MAI03670
C                                                                         * MAI03680
C         WRITE AIRCRAFT CONSTANTS                                        * MAI03690
C                                                                         * MAI03700
      WRITE(6,2050)TYPE                                                     MAI03710
      WRITE(6,2060)$MASS,CBAR,AREA,$IX,$IY,$IZ,GRAV,RHO                     MAI03720
      WRITE(6,2070)AILDEF,RUDDEF,ELEDEF                                     MAI03730
      IF(NSFLAG.EQ.4) GO TO 156                                             MAI03740
C                                                                         * MAI03750
C         WRITE PROGRAM CONSTANTS (INTEGRATION SOLUTION METHOD ONLY)      * MAI03760
C                                                                         * MAI03770
      WRITE(6,2080)EPS,DELT,LIMIT                                           MAI03780
      WRITE(6,2090)SCF                                                      MAI03790
C                                                                         * MAI03800
C         WRITE SOLUTION METHOD                                           * MAI03810
```

```
C                                                                          * MAI03820
C                                                                          *
      IF((NSFLAG.EQ.0).OR.(NSFLAG.EQ.2))  WRITE(6,2100)                      MAI03830
      IF((NSFLAG.EQ.1).OR.(NSFLAG.EQ.3)) WRITE(6,2110)                       MAI03840
  156 IF(NSFLAG.EQ.4) WRITE(6,2115)                                          MAI03850
C                                                                          * MAI03860
C         CONVERT FROM INPUT UNITS/VARIABLES TO EQUATION COMPATIBLE        * MAI03870
C         UNITS/VARIABLES                                                  * MAI03880
C                                                                          * MAI03890
      CALL EULER(1)                                                          MAI03900
C                                                                          * MAI03910
C         COMPUTE ALPHA AND BETA FROM INITIAL CONDITIONS                   * MAI03920
C                                                                          * MAI03930
      CALL EQNS(0,& 160)                                                     MAI03940
C                                                                          * MAI03950
C         CONVERT EQUATION UNITS/VARIABLES BACK TO OUTPUT VARIABLES/       * MAI03960
C         UNITS                                                            * MAI03970
C                                                                          * MAI03980
      CALL EULER(0)                                                          MAI03990
C                                                                          * MAI04000
C         WRITE INITIAL CONDITIONS AND COMPUTED ANGLE OF ATTACK AND        * MAI04010
C         SIDE SLIP ANGLE                                                  * MAI04020
C                                                                          * MAI04030
      WRITE(6,2120)ICCODE                                                    MAI04040
      WRITE(6,2130)(R(I),I=1,6),ALF,BET                                      MAI04050
      IF((NSFLAG.EQ.0).OR.(NSFLAG.EQ.2)) WRITE(6,2140) R(13)                 MAI04060
      IF((NSFLAG.EQ.1).OR.(NSFLAG.EQ.3)) WRITE(6,2150)(R(13),I=13,16)        MAI04070
C                                                                          * MAI04080
C         CONVERT UNITS                                                    * MAI04090
C                                                                          * MAI04100
      CALL EULER(1)                                                          MAI04110
C                                                                          * MAI04120
C         CHECK ALPHA AND BETA TO SEE IF THEY EXCEED LIMIT OF DATA         * MAI04130
C         DEFINITION. IF SO, A MESSAGE WILL BE WRITTEN BY ANGLE AND        * MAI04140
C         THE PROGRAM WILL ATTEMPT TO READ THE NEXT DATA SET OF INI-       * MAI04150
C         TIAL CONDITIONS DEPENDING ON THE VALUE OF NSFLAG                 * MAI04160
C                                                                          * MAI04170
      CALL ANGLE(& 160)                                                      MAI04180
C                                                                          * MAI04190
C         NOW FIND A SOLUTION                                              * MAI04200
C                                                                          * MAI04210
      IF(NSFLAG.LE.3) CALL SOLVIT                                            MAI04220
      IF(NSFLAG.EQ.4) CALL GAWAIN                                            MAI04230
C                                                                          * MAI04240
C         FAIL SAFE STATEMENT                                              * MAI04250
C                                                                          * MAI04260
      IF(NSFLAG.GE.5) WRITE(6,2160)                                          MAI04270
C                                                                          * MAI04280
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C         GO BACK AND READ THE NEXT INITIAL CONDITION DATA SET          * MAI04290
C                                                                       * MAI04300
  160 CONTINUE                                                            MAI04310
  170 STOP                                                                MAI04320
 1000 FORMAT(A4,2I3,7F10.2/(8F10.2))                                      MAI04330
 1001 FORMAT(I10,10X,A4,10X,I6)                                           MAI04340
 1002 FORMAT(8F10.6)                                                      MAI04350
 1003 FORMAT(2F10.6,I10,6F5.2,2I10)                                       MAI04360
 1004 FORMAT(7F10.6,I10)                                                  MAI04370
 2000 FORMAT(/////'0',T58,A4,/'0',T52,A4,'(ALPHA,BETA)'//                 MAI04380
     1' ','BETA: ', 9(3X,I3,5X)/  '0','ALPHA' )                           MAI04390
 2010 FORMAT(' ',I3,10(1X,F10.6))                                         MAI04400
 2020 FORMAT('0',T51,A4,' F(ALPHA ONLY)'//                                MAI04410
     1' ','ALPHA',3X,10(A4,7X))                                           MAI04420
 2030 FORMAT(////'0',T32,'THE BELOW LISTED COEFFICIENTS HAVE BEEN EQUATEMAI04430
     1D TO ZERO',///)                                                     MAI04440
 2040 FORMAT('0',T34,(7(A4,4X)))                                          MAI04450
 2050 FORMAT('1',T44,'INITIAL INPUT AND COMPUTED DATA'/' ',T49,'FOR THE MAI04460
     1',A4,' AIRCRAFT'///   ' ',T8,'MASS',T22,'CHORD',T35,'WING AREA',T5MAI04470
     24,'IX',T69,'IY',T84,'IZ',T96,'GRAVITY',T111,'DENSITY'/              MAI04480
     3' ',T7,'(SLUGS)',T21,'(FEET)',T35,'(FEET**2)',T50,'            (SMAI04490
     4LUG*FEET**2)           ',T94,'(FT/SEC**2)',T107,'(SLUGS/FT**3)') MAI04500
 2060 FORMAT('0',8(E13.4,2X)///)                                          MAI04510
 2070 FORMAT('0',T50,'CONTROL DEFLECTIONS'/  ' ',T55,'(DEGREES)'///' ', MAI04520
     1T36,'AILERON',T57,'RUDDER',T76,'ELEVATOR'// ' ',T37,F6.2,T57,F6.2,MAI04530
     2T77,F6.2/////)                                                      MAI04540
 2080 FORMAT('0',T50,'SOLUTION PARAMETERS',///                            MAI04550
     1' ',T37,'EPSILON',T58,'DELTA T',T72,'ITERATION LIMIT',/             MAI04560
     2'0',T37,F7.4,T58,F7.4,T78,I4/////)                                  MAI04570
 2090 FORMAT('0',T46,'SOLUTION CONVERGENCE FACTORS',///                   MAI04580
     1' ',T3,'ALTITUDE RATE',T27,'RADIUS',T46,'SPINRATE',T67,'THETA',   MAI04590
     2T88,'PSI',T108,'PHI'/                                               MAI04600
     3'0',T8,6(F4.2,16X)/////)                                            MAI04610
 2100 FORMAT('0',T42,'SOLUTION METHOD:  MODIFIED EQUATIONS'/////)         MAI04620
 2110 FORMAT('0',T44,'SOLUTION METHOD:  FULL EQUATIONS'/////)             MAI04630
 2115 FORMAT('0',T47,'SOLUTION METHOD:  GRADIENT'/////)                   MAI04640
 2120 FORMAT('0',T46,'INITIAL CONDITIONS (#',I6,')'///                    MAI04650
     2' ',T2,'SINK RATE',T16,'SPIN RADIUS',T32,'SPIN RATE',T49,'THETA', MAI04660
     3T65,'PSI',T80,'PHI',T94,'ALPHA',T109,'BETA'/                        MAI04670
     4' ',T3,'(FT/SEC)',T19,'(FEET)',T32,'(DEG/SEC)',T49,'(DEG)',T64,   MAI04680
     5'(DEG)',T79,'(DEG)',T94,'(DEG)',T109,'(DEG)'//)                     MAI04690
 2130 FORMAT(' ',8(F7.2,8X)//)                                            MAI04700
 2140 FORMAT('0',T55,'RADIUS RATE',/' ',T56,'(FT/SEC)'/'0',T55,F7.2)   MAI04710
 2150 FORMAT('0',T 9,'RADIUS RATE',T40,'THETA RATE',T70,'PSI RATE',T101,MAI04720
     1'PHI RATE',/' ',T11,'(FT/SEC)',T40,'(RAD/SEC)',T70,'(RAD/SEC)',T10MAI04730
     21,'(RAD/SEC)'/'0',T12,4(F7.2,13X))                                  MAI04740
 2160 FORMAT('0',T53,'NSFLAG IS GREATER THAN FOUR')                       MAI04750
      END                                                                 MAI04760
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C*********************************************************************** GAW00010
C                                                                       * GAW00020
C          SUBROUTINE GAWAIN                                            * GAW00030
C                                                                       * GAW00040
C          PURPOSE:                                                     * GAW00050
C               SOLVES SYSTEM OF NON LINEAR EQUATIONS USING THE OPTIMUM* GAW00060
C               GRADIENT METHOD.                                        * GAW00070
C                                                                       * GAW00080
C          METHOD:                                                      * GAW00090
C               A CRITERION FUNCTION IS COMPUTED AS THE 'SUM' OF THE   * GAW00100
C               ABSOLUTE VALUES OF THE EQUATION RESIDUALS. A GRADIENT  * GAW00110
C               OF THE CRITERION FUNCTION IS THEN COMPUTED WITH RESPECT* GAW00120
C               TO CHANGES IN EACH OF THE INDEPENDENT VARIABLES WHICH  * GAW00130
C               ARE SUBSEQUENTLY INCREMENTED SO AS PROVIDE A SOLUTION  * GAW00140
C               PATH WHICH WILL MINIMIZE 'SUM'. THE ITERATIVE PROCEDURE* GAW00150
C               IS CONTINUED UNTIL 'SUM' IS LESS THAN EPS OR UNTIL THE * GAW00160
C               SOLUTION PATH EXCEEDS THE DATA DEFINITION RANGE OF     * GAW00170
C               ALPHA OR BETA.                                          * GAW00180
C                                                                       * GAW00190
C          USAGE:                                                       * GAW00200
C               CALL GAWAIN                                             * GAW00210
C                                                                       * GAW00220
C          DESCRIPTION OF PARAMETERS:                                   * GAW00230
C               SUM       CRITERION FUNCTION: SUM OF THE ABSOLUTE      * GAW00240
C                         VALUES OF THE EQUATION RESIDUALS             * GAW00250
C               FAC       VALUE EQUAL TO 5*DELX(I); USED TO DETERMINE  * GAW00260
C                         GRADIENT OF SUM                               * GAW00270
C               PRSX(I):  PARTIAL DERIVATIVE OF SUM WITH RESPECT TO FAC* GAW00280
C               DELX(I):  INCREMENT COMPUTED AS A FUNCTION OF THE GRA- * GAW00290
C                         DIENT OF SUM; ENSURES A CHANGE IN THAT PARTI-* GAW00300
C                         CULAR INDEPENDENT VARIABLE WHICH WILL YIELD  * GAW00310
C                         A SUBSEQUENT REDUCTION IN SUM.               * GAW00320
C               K:        COEFFICIENT USED TO CONTROL THE INCREMENT    * GAW00330
C                         STEP SIZE                                     * GAW00340
C               M2:       INTEGER EQUAL TO THE ITERATION NUMBER WHEN   * GAW00350
C                         THE MOST RECENT COMPUTATION OF THE GRADIENT  * GAW00360
C                         OCCURRED                                      * GAW00370
C               M1:       ITERATION NUMBER                              * GAW00380
C                                                                       * GAW00390
C          COMMON VARIABLES UTILIZED (DESCRIBED IN MAIN PROGRAM)        * GAW00400
C               R               VEL                                     * GAW00410
C               ALF/BET                                                 * GAW00420
C               EPS                                                     * GAW00430
C                                                                       * GAW00440
C          SUBROUTINES REQUIRED                                         * GAW00450
C               EQNS                                                    * GAW00460
C               EULER                                                   * GAW00470
C*********************************************************************** GAW00490
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      SUBROUTINE GAWAIN                                                 GAW00500
      IMPLICIT REAL*8(A-H,O-Z,$)                                        GAW00510
      COMMON/WORKA/$MASS,RHO,CBAR,GRAV,AREA,$IX,$IY,$IZ,PI,IMAX,JMAX,   GAW00520
     1A(22),B(9),CFORCM(22,9,22),KEY(42),AILDEF,RUDDEF,ELEDEF           GAW00530
      COMMON/WORKB/ALPHA,ALF,BETA,BET,R(16),DELT,SCF(6),LIMIT,NSFLAG,   GAW00540
     1EPS,SUM ,VEL                                                      GAW00550
      DIMENSION      DELX(6),PRSX(6)                                    GAW00560
      REAL K                                                            GAW00570
C                                                                     * GAW00580
C         ENSURE THAT THE DERIVATIVES ARE EQUAL TO ZERO: EXTRANEOUS   * GAW00590
C         VALUES WILL YIELD ERRONEOUS VALUES OF ALPHA AND BETA IN     * GAW00600
C         SUBROUTINE EQNS.                                            * GAW00610
C                                                                     * GAW00620
      DO 5 I=13,16                                                      GAW00630
    5 R(I)=0.0                                                          GAW00640
      WRITE(6,998)                                                      GAW00650
C                                                                     * GAW00660
C         INITIALIZE DELX(I)                                          * GAW00670
C                                                                     * GAW00680
      DO 10 I=1,6                                                       GAW00690
 10   DELX(I)=0.01*R(I)                                                 GAW00700
      SUM=1.0                                                           GAW00710
C                                                                     * GAW00720
C         ITERATION DO LOOP                                           * GAW00730
C                                                                     * GAW00740
      DO 60 M1=1,LIMIT                                                  GAW00750
      SUMOLD=SUMALL                                                     GAW00760
C                                                                     * GAW00770
C         COMPUTE CRITERION FUNCTION                                  * GAW00780
C                                                                     * GAW00790
      CALL EQNS(M,&70)                                                  GAW00800
      SUMALL=SUM                                                        GAW00910
C                                                                       GAW00810
C         WRITE ITERATION VALUES                                        GAW00820
C                                                                       GAW00830
      CALLEULER(0)                                                      GAW00840
      WRITE(6,999)M1,(R(I),I=1,6),ALF,BET,(R(I),I=7,12),SUM,VEL         GAW00850
      CALL EULER(1)                                                     GAW00860
C                                                                     * GAW00870
C         CHECK FOR CONVERGENCE/SOLUTION                              * GAW00880
C                                                                     * GAW00890
      IF(SUM.LE.EPS) GO TO 70                                           GAW00900
C                                                                     * GAW00920
C         IF PREVIOUS SUM IS GREATER THAN CURRENT SUM THE GRADIENT IS * GAW00930
C         STILL NEGATIVE THERFORE DELX(I) WILL NOT BE RECOMPUTED. IF, * GAW00940
C         HOWEVER THE GRADIENT IS POSITIVE RECOMPUTE DELX(I).         * GAW00950
```

```
      IF(SUMOLD-SUM)20,50,50                                            GAW00970
   20 M2=M1                                                             GAW00980
C                                                                     * GAW00990
C          COMPUTE PARTIAL DERIVATIVES                                * GAW01000
C                                                                     * GAW01010
      DO 30 I=1,6                                                       GAW01020
      FAC=5.0*DELX(I)                                                   GAW01030
C     FAC=DELX(I)                                                       GAW01040
C     FAC=R(I)*0.01                                                     GAW01050
      R(I)=R(I) +FAC                                                    GAW01060
      CALL EQNS(M,&70)                                                  GAW01070
      R(I)=R(I) -FAC                                                    GAW01080
      PRSX(I)=(SUM  -SUMALL)/FAC                                        GAW01090
   30 CONTINUE                                                          GAW01100
C                                                                     * GAW01110
C          COMPUTE K                                                  * GAW01120
C                                                                     * GAW01130
      SUMSQR=0.0                                                        GAW01140
      DO 40 I=1,6                                                       GAW01150
      SUMSQR=SUMSQR + PRSX(I)**2                                        GAW01160
   40 CONTINUE                                                          GAW01170
      K=-SUMALL/SUMSQR                                                  GAW01180
C                                                                     * GAW01190
C          WRITE ITERATION VALUES                                     * GAW01200
C                                                                     * GAW01210
   50 CALL EULER(0)                                                     GAW01220
      WRITE(6,1000) M2, DELX,PRSX,K                                     GAW01230
      CALL EULER(1)                                                     GAW01240
C                                                                     * GAW01250
C          COMPUTE DELX(I)                                            * GAW01260
C          INCREMENT INDEPENDENT VARIABLE, R(I)                       * GAW01270
C                                                                     * GAW01280
      DO 60 I=1,6                                                       GAW01290
      DELX(I) =K*PRSX(I)                                                GAW01300
      R(I)=R(I) +DELX(I)                                                GAW01310
   60 CONTINUE                                                          GAW01320
   70 RETURN                                                            GAW01330
  998 FORMAT('1','M1',    T5,'ALTITUDE RATE',T26,'RADIUS',T42,'SPIN RATE'GAW01340
     1,T61,'THETA',T79,'PSI',T96,'PHI',T112,'ALPHA',T129,'BETA'/        GAW01350
     2' ',T3,'M2',     T8 ,'(FT/SEC)',T26,'(FEET)',T42,'(DEG/SEC)',T61,  GAW01360
     3'(DEG)',T78,'(DEG)',T95,'(DEG)',T112,'(DEG)',T129,'(DEG)'/        GAW01370
     4' ',T18,6('RESID',12X),T121,'SUM'/                                GAW01380
     5' ',T18,5('DELX',13X), 'DELX ',T119,'VELOCITY'/                   GAW01390
     6' ',T18,5('PRSX',13X),'PRSX',T122,'K')                            GAW01395
  999 FORMAT('0',I4,2X,F7.2,7(10X,F7.2)/                                GAW01400
     1' ',14X,E9.3,6(8X,E9.3)/' ',T118,E9.3)                            GAW01410
 1000 FORMAT('+',4X,I2,8X,E9.3,5(8X,E9.3)/' ',14X,E9.3,6(8X,E9.3))      GAW01430
      END                                                               GAW01440
```

```
C**********************************************************************
C                                                                     *
C        SUBROUTINE SOLVIT                                            *
C                                                                     *
C        PURPOSE                                                      *
C           SOLVES EQUATIONS USING INTEGRATION METHOD                 *
C                                                                     *
C        METHOD                                                       *
C           ITERATIVE SOLUTION WHEREBY THE INDEPENDENT VARIABLE      *
C           ACCELERATIONS ARE COMPUTED FROM THE GIVEN INITIAL        *
C           CONDITIONS.  NEW INITIAL CONDITIONS ARE DERIVED BY       *
C           INTEGRATING THE ACCELERATIONS AND RATES. A SOLUTION      *
C           IS OBTAINED WHEN THE ACCELERATIONS REACH A MINIMUM       *
C           VALUE                                                     *
C                                                                     *
C        APPLICABILITY                                                *
C           FULL AND MODIFIED EQUATIONS ONLY                          *
C                                                                     *
C        USAGE                                                        *
C           CALL SOLVIT (INITIAL CONDITIONS MUST HAVE BEEN CON-       *
C           VERTED TO EQUATION COMPATIBLE FORM PRIOR TO THE CALL;     *
C           (SEE SUBROUTINE EULER)                                    *
C                                                                     *
C        DESCRIPTION OF PARAMETERS                                    *
C           SUMMIN:  MINIMUM SUM COMPUTED BY EQNS                     *
C           M1:      ITERATION DO LOOP INDEX                          *
C           MS:      INTEGER EQUAL TO THE VALUE OF M1 WHEN SUMMIN     *
C                    IS COMPUTED                                      *
C           SAVE:    VECTOR OF VALUES OF THE INDEPENDENT/DEPENDENT    *
C                    VARIABLES CORRESPONDING TO ITERATION #MS. THIS   *
C                    REPRESENTS THE BEST SOLUTION CONVERGENCE.        *
C           ZZ:      MATRIX USED TO STORE A SUMMARY OF THE ITERA-     *
C                    TIVE SOLUTION. APPROPRIATE VARIABLES ARE         *
C                    ASSIGNED TO ZZ EVERY 25 ITERATIONS               *
C                                                                     *
C        COMMON VARIABLES UTILIZED (DESCRIBED IN MAIN PROGRAM)       *
C           R               SUM               ICCODE                  *
C           ALF/BET         DELT              NFLAG                   *
C           NPRINT          SCF               LIMIT                   *
C                                                                     *
C        SUBROUTINES REQUIRED                                         *
C           ANGLE                                                     *
C           EULER                                                     *
C           EQNS                                                      *
C                                                                     *
C**********************************************************************
```

```
      SUBROUTINE SOLVIT
      IMPLICIT REAL *8 (A-H,O-Z,$)
      COMMON NFLAG,NPRINT,ICCODE
      COMMON/WORKA/$MASS,RHO,CBAR,GRAV,AREA,$IX,$IY,$IZ,PI,IMAX,JMAX,
     1A(22),B(9),CFORCM(22,9,22),KEY(42),AILDEF,RUDDEF,ELEDEF
      COMMON/WORKB/ALPHA,ALF,BETA,BET,R(16),DELT,SCF(6),LIMIT,NSFLAG,
     1EPS,SUM ,VEL
      DIMENSION ZZ(81,10),SAVE(10)
      M2=1
      SUM=0.0
      SUMMIN=1000.0
      II=1
      MM=0
C                                                                      *
C          INITIALIZE ACCELERATIONS                                    *
C                                                                      *
      DO 5 I=7,12
    5 R(I)=0.0
C                                                                      *
C          STORE INITIAL CONDITIONS IN ZZ                              *
C                                                                      *
      CALL EULER(0)
      ZZ(II,1)=FLOAT(MM)
      DO 10 J=2,7
 10   ZZ(II,J)=R(J-1)
      ZZ(II,8)=ALF
      ZZ(II,9)=BET
      ZZ(II,10)=VEL
      CALL EULER(1)
      MM=25
C                                                                      *
C          IF NPRINT = 1 SUPPRESS ITERATION PRINTOUT                   *
C                                                                      *
      IF(NPRINT.EQ.1) GO TO 20
      WRITE(6,998)
C                                                                      *
C          PRIMARY INTERATION DO LOOP                                  *
C                                                                      *
   20 DO  80 M1=1,LIMIT
C                                                                      *
C          COMPUTE ACCELERATIONS AND THE APPLICABLE RATES OF THE
C          INDEPENDENT VARIABLES
C                                                                      *
      CALL EQNS(M2,&90)
C                                                                      *
C          CHECK FOR SOLUTION                                          *
C                                                                      *
      IF(SUM.LE.EPS)GO TO 90
```

```
C                                                                      * SOL00950
C        WRITE THE INDEPENDENT VARIABLES, ASSOCIATED DERIVATIVES, AND* SOL00960
C        DEPENDENT VARIABLES ON EVERY ITERATION UNLESS NPRINT = 1     * SOL00970
C                                                                      * SOL00980
      CALL EULER(0)                                                      SOL00990
      IF(NPRINT.EQ.1) GO TO 30                                           SOL01000
      WRITE(6,999) M1,(R(I),I=1,6),ALF,BET,M2,(R(I),I=7,12),SUM,         SOL01010
     1(R(I),I=13,16),VEL                                                 SOL01020
C                                                                      * SOL01030
C        STORE VALUES OF THE SPECIFIED VARIABLES EVERY 25TH ITERATION* SOL01040
C                                                                      * SOL01050
   30 IF(M1.NE.MM) GO TO 50                                              SOL01060
      II=II+1                                                            SOL01070
      ZZ(II ,1)=FLOAT(MM)                                                SOL01080
      DO 40 J=2,7                                                        SOL01090
   40 ZZ(II,J)=R(J-1)                                                    SOL01100
      ZZ(II,8)=ALF                                                       SOL01110
      ZZ(II,9)=BET                                                       SOL01120
      ZZ(II,10)=VEL                                                      SOL01130
      MM=MM+25                                                           SOL01140
   50 IF(SUM.GT.SUMMIN)GO TO 70                                          SOL01150
C                                                                      * SOL01160
C        IF COMPUTED SUM IN LESS THAN SUMMIN, REASSIGN SUMMIN/SAVE/MS* SOL01170
C                                                                      * SOL01180
      MS=M1                                                              SOL01190
      DO 60 I=1,6                                                        SOL01200
   60 SAVE(I)=R(I)                                                       SOL01210
      SAVE(7)=R(13)                                                      SOL01220
      SAVE(8)=R(14)                                                      SOL01230
      SAVE(9)=R(15)                                                      SOL01240
      SAVE(10)=R(16)                                                     SOL01250
      SUMMIN=SUM                                                         SOL01260
   70 CALL EULER(1)                                                      SOL01270
C                                                                      * SOL01280
C        INTEGRATE DERIVATIVES/COMPUTE NEW INDEPENDENT VARIABLES      * SOL01290
C                                                                      * SOL01300
      R(1)=R(1) + R(7)*DELT*SCF(1)                                       SOL01310
      R(2)=R(2) +(R(8)*DELT**2/2 +R(13)*DELT)*SCF(2)                     SOL01320
      R(3)=R(3) + R(9)*DELT*SCF(3)                                       SOL01330
      R(4)=R(4) +(R(10)*DELT**2/2 +R(14)*DELT)*SCF(4)                    SOL01340
      R(5)=R(5)+(R(11)*DELT**2/2 +R(15)*DELT)*SCF(5)                     SOL01350
      R(6)=R(6) +(R(12)*DELT**2/2+R(16)*DELT)*SCF(6)                     SOL01360
C                                                                      * SOL01370
C        DETERMINE WHETHER CURRENT VALUES OF THETA, PSI, AND PHI HAVE* SOL01380
C        EXCEEDED 2*PI; IF SO VALUES ARE APPROPRIATELY REDUCED        * SOL01390
C                                                                      * SOL01400
      CALL ANGLE (&90)                                                   SOL01410
   80 CONTINUE                                                           SOL01420
```

```
C                                                                      * SOL01430
C         WRITE SOLUTION SUMMARY                                       * SOL01440
C                                                                      * SOL01450
   90 WRITE(6,2000) ICCODE                                               SOL01460
      DO 100 I=1,II                                                      SOL01470
      MO=IDINT(ZZ(I,1))                                                  SOL01480
  100 WRITE(6,2001)MO,(ZZ(I,J),J=2,10)                                   SOL01490
C                                                                      * SOL01500
C         WRITE BEST CONVERGENCE                                       * SOL01510
C                                                                      * SOL01520
      WRITE(6,2002)                                                      SOL01530
      WRITE(6,2003)MS,(SAVE(J),J=1,6),SUMMIN                             SOL01540
C                                                                      * SOL01550
C         WRITE MESSAGE IF THE SOLUTION WAS ABNORMALLY TERMINATED      * SOL01560
C                                                                      * SOL01570
      IF(NFLAG.EQ.2) WRITE(6,4000)ALF                                    SOL01580
      IF(NFLAG.EQ.3) WRITE(6,4001)   ALF                                 SOL01590
      IF(NFLAG.EQ.4) WRITE(6,4002)  BET                                  SOL01600
      IF(NFLAG.EQ.5) WRITE(6,4003)  BET                                  SOL01610
      IF(NFLAG.EQ.1) WRITE(6,4004)                                       SOL01620
      RETURN                                                             SOL01630
  998 FORMAT('1','M1',   T5,'ALTITUDE RATE',T26,'RADIUS',T42,'SPIN RATE'SOL01640
     1,T61,'THETA',T79,'PSI',T96,'PHI',T112,'ALPHA',T129,'BETA'/         SOL01650
     2' ',T3,'M2',    T8 ,'(FT/SEC)',T26,'(FEET)',T42,'(DEG/SEC)',T61,   SOL01660
     3'(DEG)',T78,'(DEG)',T95,'(DEG)',T112,'(DEG)',T129,'(DEG)'/         SOL01670
     4' ',T18,6('ACCEL',12X),T121,'SUM'/                                 SOL01680
     5' ',T35,'RATE',T69,2('RATE',13X),'RATE',T119,'VELOCITY')           SOL01690
  999 FORMAT('0',I4,2X,F7.2,7(10X,F7.2)/                                 SOL01700
     1' ',4X,I2,8X,E9.3,6(8X,E9.3)/                                      SOL01710
     2' ',31X,E9.3,25X,3(E9.3,8X),E9.3)                                  SOL01720
 2000 FORMAT('1',T53,'SOLUTION SUMMARY FOR IC #',I6,///                  SOL01730
     1'0',T2,'ITERATION',T13,'ALTITUDE RATE',T30,'RADIUS',T41,'SPINRATE'SOL01740
     2,T56,'THETA',T70,'PSI',T83,'PHI',T95 ,'ALPHA',T108,'BETA',T119,    SOL01750
     1'VELOCITY'/                                                        SOL01760
     3' ',T4,'NUMBER',T16,'(FT/SEC)',T30,'(FEET)',T41,'(DEG/SEC)',T56,   SOL01770
     4'(DEG)',T69,'(DEG)',T82,'(DEG)',T95 ,'(DEG)',T108,'(DEG)',T120,    SOL01780
     1'FT/SEC')                                                          SOL01790
 2001 FORMAT('0',T5,I4,T14,8(F8.2,5X),F8.2)                              SOL01800
 2002 FORMAT(' ',/////)                                                  SOL01810
 2003 FORMAT('0',T55,'SOLUTION CONVERGENCE',/'0',T33,'MINIMUM SUM OF THESOL01820
     1 ABSOLUTE VALUES OF THE INDEPENDENT VARIABLES '///'0',T2,'ITERATIOSOL01830
     2N',T15,'ALTITUDE RATE',T34,'RADIUS',T47,'SPIN RATE',T64,'THETA',   SOL01840
     3T80,'PSI',T95,'PHI',T110,'SUM'/' ',T4,'NUMBER',T18,'(FT/SEC)',T34,SOL01850
     4'(FEET)',T47,'(DEG/SEC)',T64,'(DEG)',T79,'(DEG)',T94,'(DEG)'/      SOL01860
     5'0',T6,I3,T16,7(F8.2,7X))                                          SOL01870
 3000 FORMAT(7E10.4,I10)                                                 SOL01880
 4000 FORMAT('0',T35,'SOLUTION TERMINATED WHEN ALPHA EXCEEDED UPPER LIMISOL01890
     1T (',F5.1,' DEGREES)')                                             SOL01900
```

```
4001 FORMAT('0',T35,'SOLUTION TERMINATED WHEN ALPHA EXCEEDED LOWER LIMISOL01910
    1T (',F5.1,' DEGREES)')                                            SOL01920
4002 FORMAT('0',T35,'SOLUTION TERMINATED WHEN BETA EXCEEDED UPPER LIMITSOL01930
    1 (',F5.1,' DEGREES)')                                             SOL01940
4003 FORMAT('0',T35,'SOLUTION TERMINATED WHEN BETA EXCEEDED LOWER LIMITSOL01950
    1 (',F5.1,' DEGREES)')                                             SOL01960
4004 FORMAT('0',T32,'SOLUTION TERMINATED DUE TO OVERFLOW, UNDERFLOW, ORSOL01970
    1 DIVISION BY ZERO')                                               SOL01980
     END                                                               SOL01990
```

```
C*****************************************************************************
C                                                                            *
C        SUBROUTINE EQNS                                                     *
C                                                                            *
C        PURPOSE                                                             *
C           COMPUTES RESIDUALS/ACCELERATIONS OF APPROPRIATE EQUA-            *
C           TIONS AS DETERMINED BY NSFLAG                                    *
C                                                                            *
C        USAGE                                                               *
C           CALL EQNS(M,&____)                                               *
C                                                                            *
C        DESCRIPTION OF PARAMETERS                                           *
C           C:     VECTOR OF INTERPOLATED VALUES OF STABILITY                *
C                  DERIVATIVES                                               *
C           M:     CALLING ARGUMENT WHICH IF EQUAL TO 0 WILL                 *
C                  INITIATE A RETURN TO THE CALLING PROGRAM AFTER            *
C                  THE COMPUTATION OF ALPHA/BETA. ALSO USED AS               *
C                  COUPLED EQUATIONS DO LOOP INDEX                           *
C           &___:  CORRESPONDS TO THE STATEMENT NUMBER IN THE CALL-          *
C                  ING PROGRAM TO WHICH THE SUBROUTINE WILL RETURN           *
C                  IN THE EVENT OF A RETURN 1                                *
C           NFLAG  SET EQUAL TO 1 BY SUBROUTINE FIX WHENEVER THE             *
C                  FORTRAN EXTENDED ERROR HANDLING FACILITY DETECTS          *
C                  AN EXPONENT UNDERFLOW, OVERFLOW OR ATTEMPT TO             *
C                  DIVIDE BY ZERO. WHEN NFLAG = 1 IS DETECTED, A             *
C                  RETURN 1 IS EXECUTED AND THE SOLUTION IS                  *
C                  TERMINATED                                                *
C                                                                            *
C        COMMON VARIABLES UTILIZED (EXPLAINED IN MAIN PROGRAM)               *
C           $MASS    AREA    AILDEF    BETA      EPS                         *
C           RHO      $IX     RUDDEF    R         SUM                         *
C           CBAR     $IY     ELEDEF    SCF       VEL                         *
C           GRAV     $IZ     ALPHA     NSFLAG    NFLAG                       *
C                                                                            *
C        SUBROUTINES REQUIRED                                                *
C           ANGLE                                                            *
C           INTER                                                            *
C           FIX                                                              *
C                                                                            *
C*****************************************************************************

      SUBROUTINE EQNS(M,*)
      IMPLICIT REAL *8 (A-H,O-Z,$)
      EXTERNAL FIX
      COMMON NFLAG
      COMMON/WORKA/$MASS,RHO,CBAR,GRAV,AREA,$IX,$IY,$IZ,PI,IMAX,JMAX,
```

```
     1A(22),B(9),CFJRCM(22,9,22),KEY(42),AILDEF,RUDDEF,ELEDEF               EQN00470
      COMMON/WORKB/ALPHA,ALF,BETA,BET,R(16),DELT,SCF(6),LIMIT,NSFLAG,      EQN00480
     1EPS,SUM ,VEL                                                         EQN00490
      DIMENSION  C(42)                                                     EQN00500
      NFLAG =0                                                             EQN00510
C                                                                        * EQN00520
C         COMPUTE THE DIRECTION COSINES                                  * EQN00530
C                                                                        * EQN00540
      A11=DCOS(R(6))*DCOS(R(5))-DSIN(R(6))*DSIN(R(5))*DCOS(R(4))           EQN00550
      A12=DCOS(R(6))*DSIN(R(5))+DSIN(R(6))*DCOS(R(5))*DCOS(R(4))           EQN00560
      A13=DSIN(R(4))*DSIN(R(6))                                            EQN00570
      A21=-DSIN(R(6))*DCOS(R(5))-DCOS(R(6))*DSIN(R(5))*DCOS(R(4))          EQN00580
      A22=-DSIN(R(6))*DSIN(R(5))+DCOS(R(6))*DCOS(R(5))*DCOS(R(4))          EQN00590
      A23=DSIN(R(4))*DCOS(R(6))                                            EQN00600
      A31=DSIN(R(4))*DSIN(R(5))                                            EQN00610
      A32=-DSIN(R(4))*DCOS(R(5))                                           EQN00620
      A33=DCOS(R(4))                                                       EQN00630
C                                                                        * EQN00640
C         COMPUTE ANGLE OF ATTACK AND SIDESLIP ANGLE                     * EQN00650
C                                                                        * EQN00660
      ALPHA= DATAN((R(3)*R(2)*A32 + R(1)*A33 + R(13)*A31)                  EQN00670
     1/(R(3)*R(2)*A12 + R(1)*A13 + R(13)*A11))                             EQN00680
      BETA= DATAN((R(3)*R(2)*A22 + R(1)*A23 +R(13)*A21)                    EQN00690
     1/(R(3)*R(2)*A12 + R(1)*A13 + R(13)*A11))                             EQN00700
C                                                                        * EQN00710
C         RETURN IF CALL IS FROM MAIN                                    * EQN00720
C                                                                        * EQN00730
      IF(M.EQ.0)RETJRN                                                     EQN00740
C                                                                        * EQN00750
C         CHECK ALPHA/BETA LIMITS                                        * EQN00760
C                                                                        * EQN00770
      CALL ANGLE(&90)                                                      EQN00780
C                                                                        * EQN00790
C         INTERPOLATE STABILITY DERIVATIVES                              * EQN00800
C                                                                        * EQN00810
      DO 10 NA=1,42                                                        EQN00820
      CALL  INTER(NA,C(NA))                                                EQN00830
   10 CONTINUE                                                             EQN00840
C                                                                        * EQN00850
C         COMPUTE FORCE AND MOMENT COEFFICIENTS                          * EQN00860
C                                                                        * EQN00870
      VEL=DSQRT(R(1)**2 + (R(3)*R(2))**2 + R(13)**2)                       EQN00880
      FAC13=((R(15)+R(3))*A13+R(14)*DCOS(R(6)))*CBAR/2.0/VEL               EQN00890
      FAC23=((R(15)+R(3))*A23-R(14)*DSIN(R(6)))*CBAR/2.0/VEL               EQN00900
      FAC33=((R(15)+R(3))*A33 +R(16))*CBAR/2.0/VEL                         EQN00910
      CFX=C(1) +C(25)*FAC13 + C(26)*FAC23 + C(23)*FAC33                    EQN00920
     1+C(27)*AILDEF + C(24)*RUDDEF + C( 7)*ELEDEF                          EQN00930
      CFY=C(2) +C(8 )*FAC13 + C(28)*FAC23 + C(9)*FAC33                     EQN00940
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      1+C(10)*AILDEF + C(11)*RUDDEF + C(29)*ELEDEF
       CFZ=C(3) +C(30)*FAC13 + C(31)*FAC23 + C(32)*FAC33
      1+C(33)*AILDEF + C(34)*RUDDEF + C(12)*ELEDEF
       CMX=C(4) + C(13)*FAC13+ C(35)*FAC23 + C(14)*FAC33
      1+C(15)*AILDEF + C(16)*RUDDEF + C(36)*ELEDEF
       CMY=C(5) +C(37)*FAC13 + C(17)*FAC23 + C(38)*FAC33
      1+C(39)*AILDEF + C(40)*RUDDEF + C(18)*ELEDEF
       CMZ=C(6) +C(19)*FAC13 + C(41)*FAC23 + C(20)*FAC33
      1+C(21)*AILDEF + C(22)*RUDDEF + C(42)*ELEDEF
       FAC1=((R(3)*R(2))**2 + R(1)**2 + R(13)**2)
       FAC2=RHO * AREA / 2.0/$MASS
       IF(NSFLAG.NE.4) GO TO 30
C                                                                      *
C           SHORT EQUATIONS                                            *
C                                                                      *
       R(7)=-GRAV-(CFX*A13 +CFY*A23 + CFZ*A33)*FAC2*FAC1
       R(8)=R(3)**2*R(2)-(CFX*A11 + CFY*A21 + CFZ*A31)*FAC2*FAC1
       R(9)=CFX*A12+CFY*A22+CFZ*A32
       R(10)=2.0*($IX*(DSIN(R(6))**2)+$IY*(DCOS(R(6))**2)-$IZ)*R(3)**2
      1*DSIN(R(4))*DCOS(R(4))+FAC1*RHO*AREA*CBAR
      2*(CMX*DCOS(R(6))-CMY*DSIN(R(6)))
       R(11)=CMX*A13+CMY*A23+CMZ*A33
       R(12)=2.0*($IX-$IY)*R(3)**2*(DSIN(R(4))**2)*DSIN(R(6))*DCOS(R(6)
      1)+RHO*AREA*CBAR*FAC1*CMZ
C                                                                      *
C           COMPUTE SUM OF THE ABSOLUTE VALUES OF THE RESIDUALS        *
C                                                                      *
       SUM=0.0
       DO 20 I=7,12
    20 SUM=SUM+DABS(R(I))
       RETURN
C                                                                      *
C           VERTICAL AND RADIAL EQUATIONS (SAME FOR FULL AND MODIFIED  *
C           FORM                                                       *
C                                                                      *
    30 R(7)=-GRAV-(CFX*A13 +CFY*A23 +CFZ*A33)*FAC2*FAC1
       IF(SCF(1).EQ.0.0) R(7)=0.0
       R(8)=R(3)**2*R(2)-(CFX*A11 + CFY*A21 + CFZ*A31)*FAC2*FAC1
       IF(SCF(2).EQ.0.0)R(8)=0.0
C                                                                      *
C           COMPUTE NEW RADIUS RATE                                    *
C                                                                      *
       R(13)=(R(13) +R(8)*DELT)*SCF(2)
       FAC1=((R(3)*R(2))**2 + R(1)**2 + R(13)**2)
       SUM1=1.0D+25
       IF(NFLAG.EQ.1) RETURN 1
C                                                                      *
C                                                                      *
```

```
C         DO LOOP TO STABILIZE COUPLED ACCELERATIONS                      * EQN01420
C                                                                         * EQN01430
      DO 60 M=1,20                                                          EQN01440
      IF(NFLAG.EQ.1) RETURN 1                                               EQN01450
      IF((NSFLAG.EQ.1).OR.(NSFLAG.EQ.3))GO TO 40                            EQN01460
C                                                                         * EQN01470
C         MODIFIED EQUATIONS                                              * EQN01480
C                                                                         * EQN01490
      R(9)=((-$IX*(R(11)*A13 + R(10)*DCOS(R(6)))*A13                        EQN01500
     1-$IY*(R(11)*A23 - R(10)*DSIN(R(6)))*A23                               EQN01510
     2-$IZ*(R(11)*A33+R(12))*A33) -2.0*$MASS*R(2)*R(3)*R(13)                EQN01520
     3-(R(2)*(CFX*A12 + CFY*A22 + CFZ*A32)                                  EQN01530
     4- CBAR*(CMX*A13 + CMY*A23 + CMZ*A33))*FAC2*$MASS*FAC1)                EQN01540
     5/($MASS*R(2)**2 + $IX*A13**2 + $IY*A23**2+$IZ*A33**2)                 EQN01550
      IF(SCF(3).EQ.0.0) R(9)=0.0                                            EQN01560
      R(10)=(-(R(11) + R(9))*($IX*A13*DCOS(R(6))-$IY*A23*DSIN(R(6)))       EQN01570
     1+ $IX*(R(3)**2)*A13*DCOS(R(4))*DSIN(R(6))                             EQN01580
     2+$IY*(R(3)**2)*A23*DCOS(R(4))*DCOS (R(6))                             EQN01590
     3-$IZ*(R(3)**2)*A33*DSIN(R(4))                                         EQN01600
     4+(CMX*DCOS(R(6))-CMY*DSIN(R(6)))                                      EQN01610
     6*FAC2*CBAR*$MASS*FAC1)                                                EQN01620
     7/($IX*DCOS(R(6))**2+$IY*DSIN(R(6)) **2)                               EQN01630
      IF(SCF(4).EQ.0.0) R(10)=0.0                                           EQN01640
      R(11)=(-$IX*(R(9)*A13 + R(10)*DCOS(R(6)))*A13                         EQN01650
     1-$IY*(R(9)*A23 - R(10)*DSIN(R(6)))*A23                                EQN01660
     2-$IZ*(R(9)*A33+R(12))*A33                                             EQN01670
     3+(CMX*A13 + CMY*A23 + CMZ*A33)                                        EQN01680
     7*FAC2*CBAR*$MASS*FAC1)                                                EQN01690
     6/($IX*A13**2 + $IY*A23**2 + $IZ*A33**2)                               EQN01700
      IF(SCF(5).EQ.0.0) R(11)=0.0                                           EQN01710
      R(12)=(($IX-$IY)*R(3)**2*A13*A23-$IZ*(R(11)+R(9))*A33                 EQN01720
     1+CMZ*FAC1*FAC2*$MASS*CBAR)/$IZ                                        EQN01730
      IF(SCF(6).EQ.0.0) R(12)=0.0                                           EQN01740
      IF((NSFLAG.EQ.0).OR.(NSFLAG.EQ.2))GO TO 50                            EQN01750
C                                                                         * EQN01760
C         FULL EQUATIONS                                                  * EQN01770
C                                                                         * EQN01780
   40 R(9)=(-2.0*($MASS*R(3)*R(2)*R(13) + (R(15)+R(3))                      EQN01790
     9*(($IX-$IY)*DCOS(R(6))*A13*R(16)                                      EQN01800
     1+($IX*DSIN(R(6))**2+ $IY*DCOS(R(6))**2-$IZ)*A33*R(14) )               EQN01810
     2*DSIN(R(4))) - R(11)*($IX*A13**2 +$IY*A23**2 + $IZ*A33**2)            EQN01820
     3-R(10)   *($IX-$IY)*A13*DCOS(R(6))-R(12)*$IZ*A33                      EQN01830
     4+(($IX-$IY)*(DSIN(R(6))**2 -DCOS(R(6))**2)+$IZ)*R(16)*R(14) *         EQN01840
     5DSIN(R(4))-($IX-$IY)*DSIN(R(6))*DCOS(R(6))*DCOS(R(4))*R(14)           EQN01850
     6**2-(R(2)*(CFX*A12 + CFY*A22 + CFZ*A32)                               EQN01860
     7- CBAR*(CMX*A13 + CMY*A23 + CMZ*A33))*FAC2*$MASS*FAC1)                EQN01870
     8/($MASS*R(2)**2 + $IX*A13**2 + $IY*A23**2+$IZ*A33**2)                 EQN01880
      IF(SCF(3).EQ.0.0)R(9)=0.0                                             EQN01890
```

```
      R(10)=((R(15)+ R(3))*($IX*DSIN(R(6))**2+$IY*DCOS(R(6))**2 -$IZ)   EQN01900
     1*(R(16) +(R(15)+R(3))*DCOS(R(4)))*DSIN(R(4))                        EQN01910
     2+2.0*($IX-$IY)*R(16)*R(14) *DSIN(R(6))*DCOS(R(6))                   EQN01920
     3-(R(15)+R(3))*($IX*DCOS(R(6))**2 + $IY*DSIN(R(6))**2)*DSIN(R(4     EQN01930
     4))*R(16)-($IX-$IY)*(R(9)+R(11))*DSIN(R(4))*DSIN(R(6))*DCOS(R(6)    EQN01940
     5)+(CMX*DCOS(R(6))-CMY*DSIN(R(6)))                                   EQN01950
     6*FAC2*CBAR*$MASS*FAC1)                                              EQN01960
     7/($IX*DCOS(R(6))**2+$IY*DSIN(R(6)) **2)                             EQN01970
      IF(SCF(4).EQ.0.0)R(10)=0.0                                          EQN01980
      R(11)=(-2.0*(R(15)+R(3))*(($IX-$IY)*A13*A23*R(16)                   EQN01990
     1+($IX*DSIN(R(6))**2+$IY*DCOS(R(6))**2-$IZ)*DSIN(R(4))*DCOS(R(4     EQN02000
     2))*R(14) )-($IX-$IY)*R(10)*A13*DCOS(R(6))-$IZ*R(12)*A33             EQN02010
     3-R(9)*($IX*A13**2+$IY*A23**2+$IZ*A33**2)                            EQN02020
     4+(($IX-$IY)*(DSIN(R(6))**2-DCOS(R(6))**2)+$IZ)*DSIN(R(4))*R(16)     EQN02030
     5*R(14) -($IX-$IY)*DSIN(R(6))*DCOS(R(6))*DCOS(R(4))*R(14) **2        EQN02040
     3+(CMX*A13 + CMY*A23 + CMZ*A33)                                      EQN02050
     7*FAC2*CBAR*$MASS*FAC1)                                              EQN02060
     8/($IX*A13**2 + $IY*A23**2 + $IZ*A33**2)                             EQN02070
      IF(SCF(5).EQ.0.0)R(11)=0.0                                          EQN02080
      R(12)=(-$IZ*(R(11)+R(9))*A33 + ($IX-$IY)*((R(15)+R(3))*A13          EQN02090
     1+R(14)*DCOS(R(6)))*((R(15)+R(3))*A23-R(14)*DSIN(R(6)))              EQN02100
     2+CMZ*FAC1*FAC2*$MASS*CBAR)/$IZ                                      EQN02110
      IF(SCF(6).EQ.0.0)R(12)=0.0                                          EQN02120
   50 SUM2=SUM1                                                           EQN02130
      SUM1=DABS(R(9))+DABS(R(10))+DABS(R(11))+DABS(R(12))                 EQN02140
C                                                                       * EQN02150
C        IF THE SUM OF THE ABSOLUTE VALUES OF THE COUPLED ACCELERA-     * EQN02160
C        TIONS DIFFERS FROM THAT OF THE PREVIOUS ITERATION BY LESS      * EQN02170
C        THAN EPS, JUMP OUT OF DO LOOP                                  * EQN02180
C                                                                       * EQN02190
      IF(DABS(SUM2-SUM1).LE.EPS) GO TO 70                                 EQN02200
   60 CONTINUE                                                            EQN02210
C                                                                       * EQN02220
C        COMPUTE SUM OF ALL ACCELERATIONS                               * EQN02230
C                                                                       * EQN02240
   70 SUM=SUM1 +DABS(R(7)) +DABS(R(8))                                    EQN02250
      IF(NFLAG.EQ.1) RETURN 1                                             EQN02260
      IF((NSFLAG.EQ.0).OR.(NSFLAG.EQ.2))GO TO 80                          EQN02270
C                                                                       * EQN02280
C        COMPUTE ORIENTATION ANGLE RATES                                * EQN02290
C                                                                       * EQN02300
      R(14)=(R(14)+R(10)*DELT)*SCF(4)                                     EQN02310
      R(15)=(R(15) +R(11)*DELT)*SCF(5)                                    EQN02320
      R(16)=(R(16)+R(12)*DELT)*SCF(6)                                     EQN02330
   80 RETURN                                                              EQN02340
   90 RETURN 1                                                            EQN02350
      END                                                                 EQN02360
```

```
C***********************************************************************
C                                                                      *
C          SUBROUTINE ANGLE                                            *
C                                                                      *
C          PURPOSE                                                     *
C              REDUCES MAGNITUDE OF ANGULAR VARIABLES TO LESS THAN 2PI *
C              INITIATES TERMINATION OF ITERATIVE SOLUTION IF ALPHA OR *
C              BETA EXCEEDS THEIR RESPECTIVE REGIONS OF DEFINITION     *
C                                                                      *
C          USAGE                                                       *
C              CALL ANGLE(*)                                           *
C                                                                      *
C          DESCRIPTION OF PARAMETERS                                   *
C              *:   CORRESPONDS TO THE STATEMENT NUMBER IN THE CALLING *
C                   PROGRAM TO WHICH THE SUBROUTINE WILL RETURN IN THE *
C                   EVENT OF A RETURN 1                                *
C              NFLAG:  USED IN THIS SUBROUTINE TO COMMUNICATE WHICH    *
C                      A/B LIMIT HAS BEEN EXCEEDED IN THE EVENT OF AN  *
C                      ABNORMAL TERMINATION                            *
C                                                                      *
C          COMMON VARIABLES UTILIZED (EXPLAINED IN MAIN PROGRAM)     . *
C              R                                                       *
C              A; IMAX; ALPHA                                          *
C              B; JMAX; BETA                                           *
C                                                                      *
C***********************************************************************

      SUBROUTINE ANGLE (*)
      IMPLICIT REAL *8 (A-H,O-Z,$)
      COMMON NFLAG
      COMMON/WORKA/$MASS,RHO,CBAR,GRAV,AREA,$IX,$IY,$IZ,PI,IMAX,JMAX,
     1A(22),B(9),CFORCM(22,9,22),KEY(42),AILDEF,RUDDEF,ELEDEF
      COMMON/WORKB/ALPHA,ALF,BETA,BET,R(16),DELT,SCF(6),LIMIT,NSFLAG,
     1EPS,SUM ,VEL
      DIMENSION Z(5)
      PI=3.14159265358979
      Z(1)=R(4)
      Z(2)=R(5)
      Z(3)=R(6)
      Z(4)=ALPHA
      Z(5)=BETA
      DO 10 I=1,5
C
C          REDUCE TO LESS THAN THE ABSOLUTE VALUE OF 2*PI
C
      IF(Z(I))3,1,1
 1    IF(Z(I).LE.2.0 *PI)GO TO 10
```

```
      Z(I)=Z(I)-2.0*PI
      IF(Z(I).GT.100.0*PI) Z(I)=Z(I)-98.0*PI
      GO TO 1
 3    IF(Z(I).GE.-2.0*PI)GO TO 10
      Z(I)=Z(I)+2.0*PI
      IF(Z(I).LT.-100.0*PI)Z(I)=Z(I)+98.0*PI
      GO TO 3
 10   CONTINUE
C
C          CHECK WHETHER ALPHA/BETA HAS EXCEEDED UPPER/LOWER BOUNDARIES
C          OF A/B VECTOR
C
  11  IF((Z(4).GT.0).AND.(Z(4).GT.A(IMAX)))GO TO 50
 12   IF((Z(5).GT.0).AND.(Z(5).GT.B(JMAX)))GO TO 51
 13   IF((Z(4).LE.0).AND.(Z(4).LT.A(1)))GO TO 52
 14   IF((Z(5).LE.0).AND.(Z(5).LT.B(1)))GO TO 53
      GO TO 42
C
C          WRITE APPLICABLE MESSAGE AND INITIATE TERMINATION OF
C          ITERATIVE SOLUTION THROUGH RETURN 1 PROCEDURE
C
 50   WRITE(6,20)
      NFLAG=2
      ALF=ALPHA*180.0/PI
      RETURN 1
 51   WRITE(6,21)
      NFLAG=4
      BET=BETA*180.0/PI
      RETURN 1
 52   WRITE(6,40)
      NFLAG=3
      ALF=ALPHA*180.0/PI
      RETURN 1
 53   WRITE(6,41)
      NFLAG=5
      BET=BETA*180.0/PI
      RETURN 1
C
C          REASSIGN VALUES
C
 42   R(4)=Z(1)
      R(5)=Z(2)
      R(6)=Z(3)
      ALPHA=Z(4)
      BETA=Z(5)
      RETURN
 20   FORMAT('0',T16,'***** ITERATION TERMINATED: ALPHA HAS EXCEEDED THE
     1 UPPER LIMIT OF DATA DEFINITION *****')
```

```
40    FORMAT('0',T16,'***** ITERATION TERMINATED: ALPHA HAS EXCEEDED THEANG00950
     1 LOWER LIMIT OF DATA DEFINITION *****')                           ANG00960
21    FORMAT('0',T16,'***** ITERATION TERMINATED: BETA HAS EXCEEDED THE ANG00970
     1UPPER LIMIT OF DATA DEFINITION *****')                            ANG00980
41    FORMAT('0',T16,'***** ITERATION TERMINATED: BETA HAS EXCEEDED THE ANG00990
     1LOWER LIMIT OF DATA DEFINITION *****')                            ANG01000
      END                                                               ANG01010
C
C
C
C
C*********************************************************************** FIX00010
C                                                                      * FIX00020
C          SUBROUTINE FIX                                              * FIX00030
C                                                                      * FIX00040
C          PURPOSE                                                     * FIX00050
C               SETS NFLAG = 1 IN THE EVENT OF A CALL FROM THE EXTENDED* FIX00060
C               ERROR HANDLING FACILITY. A CALL TO ERRSET MUST BE MADE * FIX00070
C               IN THE MAIN PROGRAM IN ORDER TO DETERMINE WHICH ERRORS * FIX00080
C               WILL INITIATE A CALL TO FIX.                           * FIX00090
C                                                                      * FIX00100
C          DESCRIPTION OF PARAMETERS                                   * FIX00110
C               NFLAG:  INTEGER WHICH IF EQUAL TO ONE, WILL INITITATE  * FIX00120
C                       AN ABNORMAL TERMINATION TO A SOLUTION          * FIX00130
C               IRETCD: INTEGER WHICH IF EQUAL TO ZERO DIRECTS THE     * FIX00140
C                       EXTENDED ERROR HANDLING FACILITY TO EXECUTE    * FIX00150
C                       STANDARD CORRECTIVE ACTION FOR THE DETECTED    * FIX00160
C                       ERROR                                            FIX00170
C                                                                      * FIX00180
C*********************************************************************** FIX00190

      SUBROUTINE FIX(IRETCD)                                            FIX00200
      COMMON NFLAG                                                      FIX00210
      NFLAG=1,                                                          FIX00220
      IRETCD=0                                                          FIX00230
      RETURN                                                            FIX00240
      END                                                               FIX00250
```

```
C      ---------------------------------------------------------------
C
C         SUBROUTINE SPLIN
C
C         PURPOSE
C             PROVIDES INTERPOLATED VALUE USING "CUBIC SPLINE FITTING"
C
C         USAGE
C             FIRST CALL TO SUBROUTINE:
C             CALL SPLIN1(X,Y,M,XINT,YINT)
C
C             SUBSEQUENT CALLS:
C             CALL SPLINN(X,Y,M,XINT,YINT)
C
C         DESCRIPTION OF PARAMETERS
C             X:      MONOTONICALLY INCREASING ABSCISSA ARRAY
C             Y:      ONE-FOR-ONE CORRESPONDING ORDINATE ARRAY
C             M:      NUMBER OF X AND Y VALUES SUPPLIED < OR = 300
C             XINT:   VALUE OF ABSCISSA FOR WHICH CORRESPONDING ORDINATE
C                     IS TO BE INTERPOLATED (OR EXTRAPOLATED)
C             YINT:   INTERPOLATED (OR EXTRAPOLATED) ORDINATE VALUE
C
C             X,Y,XINT, AND YINT ARE REAL*8
C
C         REMARKS
C             IF SPECIFIED X FALLS OUTSIDE OF RANGE, AN EXTRAPOLATED
C             VALUE WILL BE SUPPLIED
C
C         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C             SUBROUTINE SPLICO IS INCLUDED IN SUBROUTINE SPLIN PACKAGE
C
C         MATHEMATICAL METHOD
C             UPON FIRST ENTRY TO SPLIN, A CALL TO SPLICO IS MADE TO
C             DETERMINE THE COEFFICIENTS TO BE USED IN PERFORMING THE
C             INTERPOLATIONS. SEARCH FOR BRACKETING ABSCISSA VALUES IS
C             ALWAYS MADE FROM THE REFERENCE LAST USED IN INTERPOLATING.
C
C         REFERENCE
C             PENNINGTON, RALPH H.,"INTRODUCTORY COMPUTER METHODS AND
C             NUMERICAL ANALYSIS", THE MACMILLAN COMPANY, NEW YORK, 1965
C
C         SAMPLE PROGRAMS
C
C         1.  GIVEN 10 PAIRS OF ABSCISSA AND ORDINATE VALUES:
C
C             X=1.,5.,15.,21.,25.,32.,38.,44.,75.,90.
C             Y=0.,50.,250.,400.,750.,780.,900.,950.,975.,1100.
C
```

```
C           FIND AN ORDINATE VALUE WHEN THE ABSCISSA VALUE IS:
C           20.,60., AND 100.
C
C           REAL*8 X(10),Y(10),XINT(3)/20.,60.,100./,YINT(3)
C           CALL SPLIN1(X,Y,10,XINT(1),YINT(1))
C           DO 20 I=2,3
C        20 CALL SPLINN(X,Y,10,XINT(I),YINT(I))
C
C        2. GIVEN A TWO DIMENSIONAL TABLE WITH N ROWS AND M COLUMNS,
C           N ABSCISSA VALUES, AND M ORDINATE VALUES, DETERMINE A
C           VALUE IN THE TABLE AT A SPECIFIED ABSCISSA AND ORDINATE
C           VALUE.
C
C           REAL*8 A(10,20),U(10),V(20),YY(20),YI(10),X,Y,FXY
C           N=10
C           M=20
C           DO 100 NR=1,N
C           DO 101 NC=1,M
C       101 YY(NC)=A(NR,NC)
C       100 CALL SPLIN1(V,YY,M,Y,YI(NR))
C           CALL SPLIN1(U,YI,N,X,FXY)
C
C           WHERE A CONTAINS THE TWO DIMENSIONAL TABLE
C                 N IS THE NUMBER OF ROWS IN A
C                 M IS THE NUMBER OF COLUMNS IN A
C                 U CONTAINS THE N ABSCISSA VALUES
C                 V CONTAINS THE M ORDINATE VALUES
C                 YY CONTAINS A SELECTED ROW OF THE TABLE A
C                 YI CONTAINS AN INTERPOLATED VALUE OF A FOR EACH ROW
C                    AT THE SELECTED ORDINATE VALUE
C                 X IS THE ABSCISSA FOR WHICH A FUNCTION VALUE
C                   IS INTERPOLATED
C                 Y IS THE ORDINATE FOR WHICH A FUNCTION VALUE IS
C                   TO BE RETURNED
C                 FXY IS A VALUE OF A AT X,Y
C
C
C
C
C
C     ------------------------------------------------------------------


      SUBROUTINE SPLIN1(X,Y,M,XINT,YINT)
      IMPLICIT REAL*8 (A-H),REAL*8 (O-Z)
      DIMENSION X(M),Y(M),C(4,300)
      CALL SPLICO(X,Y,M,C)
      K=1
```

```
      ENTRY SPLINN(X,Y,M,XINT,YINT)                                     SPL00950
    3 IF(XINT-X(1)) 70,1,2                                              SPL00960
   70 K=1                                                               SPL00970
      GO TO 7                                                           SPL00980
    1 YINT=Y(1)                                                         SPL00990
      RETURN                                                            SPL01000
    2 IF(XINT-X(K+1))6,4,5                                              SPL01010
    4 YINT=Y(K+1)                                                       SPL01020
      RETURN                                                            SPL01030
    5 K=K+1                                                             SPL01040
      IF(M-K) 71,71,3                                                   SPL01050
   71 K=M-1                                                             SPL01060
      GO TO 7                                                           SPL01070
    6 IF(XINT-X(K))13,12,11                                             SPL01080
   12 YINT=Y(K)                                                         SPL01090
      RETURN                                                            SPL01100
   13 K=K-1                                                             SPL01110
      GO TO 6                                                           SPL01120
    7 PRINT 101,XINT                                                    SPL01130
  101 FORMAT(8H0XINT = E18.9,32H, OUT OF RANGE FOR INTERPOLATION)       SPL01140
   11 YINT=(X(K+1)-XINT)*(C(1,K)*(X(K+1)-XINT)**2+C(3,K))               SPL01150
      YINT=YINT+(XINT-X(K))*(C(2,K)*(XINT-X(K))**2+C(4,K))              SPL01160
      RETURN                                                            SPL01170
      END                                                               SPL01180


      SUBROUTINE SPLICO(X,Y,M,C)                                        SPL01190
      IMPLICIT REAL*8 (A-H),REAL*8 (O-Z)                                SPL01200
      DIMENSION X(M),Y(M),C(4,300),D(300),P(300),E(300),A(300,3),B(300),SPL01210
     1Z(300)                                                            SPL01220
      MM=M-1                                                            SPL01230
      DO 2 K=1,MM                                                       SPL01240
      D(K)=X(K+1)-X(K)                                                  SPL01250
      P(K)=D(K)/6.                                                      SPL01260
    2 E(K)=(Y(K+1)-Y(K))/D(K)                                           SPL01270
      DO 3 K=2,MM                                                       SPL01280
    3 B(K)=E(K)-E(K-1)                                                  SPL01290
      A(1,2)=-1.-D(1)/D(2)                                              SPL01300
      A(1,3)=D(1)/D(2)                                                  SPL01310
      A(2,3)=P(2)-P(1)*A(1,3)                                           SPL01320
      A(2,2)=2.*(P(1)+P(2))-P(1)*A(1,2)                                 SPL01330
      A(2,3)=A(2,3)/A(2,2)                                              SPL01340
      B(2)=B(2)/A(2,2)                                                  SPL01350
      DO 4 K=3,MM                                                       SPL01360
      A(K,2)=2.*(P(K-1)+P(K))-P(K-1)*A(K-1,3)                           SPL01370
      B(K)=B(K)-P(K-1)*B(K-1)                                           SPL01380
      A(K,3)=P(K)/A(K,2)                                                SPL01390
    4 B(K)=B(K)/A(K,2)                                                  SPL01400
```

```
      Q=D(M-2)/D(M-1)                                                   SPL01410
      A(M,1)=1.+Q+A(M-2,3)                                              SPL01420
      A(M,2)=-Q-A(M,1)*A(M-1,3)                                         SPL01430
      B(M)=B(M-2)-A(M,1)*B(M-1)                                         SPL01440
      Z(M)=B(M)/A(M,2)                                                  SPL01450
      MN=M-2                                                            SPL01460
      DO 6 I=1,MN                                                       SPL01470
      K=M-I                                                             SPL01480
    6 Z(K)=B(K)-A(K,3)*Z(K+1)                                           SPL01490
      Z(1)=-A(1,2)*Z(2)-A(1,3)*Z(3)                                     SPL01500
      DO 7 K=1,MM                                                       SPL01510
      Q=1./(6.*D(K))                                                    SPL01520
      C(1,K)=Z(K)*Q                                                     SPL01530
      C(2,K)=Z(K+1)*Q                                                   SPL01540
      C(3,K)=Y(K)/D(K)-Z(K)*P(K)                                        SPL01550
    7 C(4,K)=Y(K+1)/D(K)-Z(K+1)*P(K)                                    SPL01560
      RETURN                                                            SPL01570
      END                                                               SPL01580
```

```
C*****************************************************************
C         SUBROUTINE EULER                                        *
C                                                                 *
C         PURPOSE                                                 *
C              CONVERTS ANGULAR VARIABLES SO AS TO BE EITHER COMPATI- *
C              BLE  WITH THE PRINTED OUTPUT OR THE APPLICABLE FORM OF *
C              THE EQUATIONS OF MOTION                            *
C                                                                 *
C         USAGE                                                   *
C              CALL EULER(N)                                      *
C                                                                 *
C         DESCRIPTION OF PARAMETERS                               *
C              N:       INTEGER WHICH DETERMINE WHICH OF TWO POSSIBLE  *
C                       DATA CONVERSIONS IS TO BE EMPLOYED        *
C              THETA:   PITCH ORIENTATION ANGLE                   *
C              PSI:     ROLL ORIENTATION ANGLE                    *
C              PHI:     YAW ORIENTATION ANGLE                     *
C                                                                 *
C                       NOTE:  REFER TO REFERENCE (A) OF THE MAIN PRO- *
C                       GRAM FOR ADDITIONAL EXPLAINATION REGARDING THE *
C                       DIFFERENCES BETWEEN THE ORDERED AND REFERENCE  *
C                       EULER ANGLES                              *
C                       N=0  EQUATION VARIABLES ARE CONVERTED FOR USE  *
C                            AS PRINTED OUTPUT  (ORDERED EULER ANGLES, *
C                            DEGREES)                             *
C                       N=1  INPUT VARIABLES ARE CONVERTED SO AS TO BE *
C                            COMPATIBLE WITH THE APPLICABLE EQUATIONS  *
C                            (REFERENCE EULER ANGLES, RADIANS)    *
C                                                                 *
C         COMMON VARIABLES UTILIZED                               *
C              ALPHA/ALF                                          *
C              BETA/BET                                           *
C              R                                                  *
C                                                                 *
C*****************************************************************

      SUBROUTINE EULER(N)
      IMPLICIT REAL *8 (A-H,O-Z,$)
      COMMON/WORKB/ ALPHA,ALF,BETA,BET,R(16),DELT,SCF(6),LIMIT,NSFLAG,
     1EPS,SUM ,VEL
      PI=3.14159265358979
C
C         N=0:  CONVERT REFERENCE EULER ANGLES TO ORDERED EULER ANGLES
C               AND THEN CONVERT ANGULAR VARIABLES FROM RADIANS TO
C               DEGREES
C
C                                                                 *
```

```
C
C         N=1:  CONVERT ANGULAR VARIABLES FROM DEGREES TO RADIANS AND
C               THEN CONVERT ORDERED EULER ANGLES TO REFERENCE EULER
C               ANGLES
C
      IF(N.EQ.1)GO TO 1
C
C         EULER ANGLE CONVERSION: REFERENCE TO ORDERED
C
      X=R(4)
      Y=R(5)
      Z=R(6)
      CTERM=DATAN(DTAN(Y/2.0)*DSIN((PI/2.0-X)/2.0)/DSIN((PI/2.0+X)/2.0))
      DTERM=DATAN(DTAN(Y/2.0)*DCOS((PI/2.0-X)/2.0)/DCOS((PI/2.0+X)/2.0))
      PSI=Z+CTERM+DTERM
      THETA=DTERM-CTERM
      PHI=PI/2.0-2.0*DATAN(DSIN(CTERM)/(DSIN(DTERM)*DTAN((PI/2.0-X)/2.0)
     1))
      R(4)=THETA*180.0/PI
      R(5)=PSI*180.0/PI
      R(6)=PHI*180.0/PI
      R(3)=R(3)*180.0/PI
      ALF=ALPHA*180.0/PI
      BET=BETA*180.0/PI
      GO TO 3
C
C         EULER ANGLE CONVERSION: ORDERED TO REFERENCE
C
    1 DO 2 I=3,6
    2 R(I)=R(I)*PI/180.0
      X=R(4)
      Y=R(5)
      Z=R(6)
      ATERM=DATAN(DTAN(X/2.0)*DTAN(Z/2.0))
      BTERM=DATAN(DTAN(X/2.0)/DTAN(Z/2.0))
      PHI=Y-ATERM-BTERM
      PSI=BTERM-ATERM
      THETA=2.0*DATAN(DSIN(ATERM)/(DSIN(BTERM)*DTAN(Z/2.0)))
      R(4)=THETA
      R(5)=PSI
      R(6)=PHI
    3 RETURN
      END
```

```
C*************************************************************************
C                                                                        *
C         SUBROUTINE INTER                                               *
C                                                                        *
C         PURPOSE                                                        *
C             INTERPOLATES COEFFICIENT MATRICES OR VECTORS               *
C                                                                        *
C         USAGE                                                          *
C             CALL INTER(NA,VALUE)                                       *
C                                                                        *
C         DESCRIPTION OF PARAMETERS                                      *
C             NA:     INTEGER CORRESPONDING TO THE COEFFICIENT MATRIX    *
C                     OR VECTOR TO BE INTERPOLATED                       *
C             VALUE:  INTERPOLATED VALUE OF THE COEFFICIENT MATRIX       *
C                                                                        *
C         COMMON VARIABLES UTILIZED (DESCRIBED IN MAIN PROGRAM)          *
C             CFORCM                                                     *
C             A; IMAX: ALPHA                                             *
C             B: JMAX: BETA                                              *
C             KEY(NA)                                                    *
C                                                                        *
C         UTILITIES/DIMENSIONS                                           *
C             AMAT(IMAX,JMAX)                                            *
C             YY(JMAX)                                                   *
C             YI(IMAX)                                                   *
C                                                                        *
C         SUBROUTINES REQUIRED                                           *
C             SUBROUTINE SPLIN                                           *
C                                                                        *
C*************************************************************************


      SUBROUTINE INTER(NA,VALUE)
      IMPLICIT REAL *8 (A-H,O-Z,$)
      COMMON/WORKA/$MASS,RHO,CBAR,GRAV,AREA,$IX,$IY,$IZ,PI,IMAX,JMAX,
     1A(22),B(9),CFORCM(22,9,22),KEY(42),AILDEF,RUDDEF,ELEDEF
      COMMON/WORKB/ALPHA,ALF,BETA,BET,R(16),DELT,SCF(6),LIMIT,NSFLAG,
     1EPS,SUM ,VEL
      DIMENSION AMAT(22,9),YY(9),YI(22)
      II=KEY(NA)+1
C
C         II=1; UNDEFINED STABILITY DERIVATIVE
C         II=2; COEFFICIENT VECTOR
C         II=3; COEFFICIENT MATRIX
C
      GO TO (1,2,4 ),II
C
```

```
C         UNDEFINED STABILITY DERIVATIVES ARE EQUATED TO ZERO
C
  1     VALUE=0.0
        RETURN
C
C           VECTOR INTERPOLATION
C
  2     DO 3 I=1,IMAX
  3     YI(I)=(CFORCM(I,1,NA))
        CALL SPLIN1(A,YI,IMAX,ALPHA,VALUE)
        RETURN
C
C           MATRIX INTERPOLATION
C
  4     DO 5  I=1,IMAX
        DO 5  J= 1,JMAX
  5     AMAT(I,J)=(CFORCM(I,J,NA))
        DO 7 NR=1,IMAX
        DO 6 NC=1,JMAX
  6     YY(NC)=AMAT(NR,NC)
C
C           USE SPLIN1 TO INTERPOLATE/FILL UTILITY VECTOR (YI) WHICH
C           CORRESPONDS TO B=BETA FOR ALL VALUES OF A
C
  7     CALL SPLIN1(B,YY,JMAX,BETA,YI(NR))
C           ASSIGN VALUES
C           NOW INTERPOLATE YI AT A=ALPHA
C
        CALL SPLIN1(A,YI,IMAX,ALPHA,VALUE)
        RETURN
        END
```

APPENDIX C

CRITERION FUNCTION SEARCH PROGRAM

The aircraft spin solution program was modified in order to evaluate the "reasonableness" of various sets of initial conditions. The resulting Criterion Function Search program is designed to compute the criterion value for a given set of initial conditions. The initial conditions are generated by six nested "do loops" within MAIN. The dependent variables α , β , V and n are subsequently computed and compared to pre-established constraints; if the constraints are satisfied, the criterion is subsequently computed. If the value is less than 10^4, the independent variables are printed. An abbreviated flowchart of the program is included as Figure C-1. Pertinent data relating to the initial condition "do loops" and dependent variable constraints is included as Table C-I. The program is basically the same as the spin solution program except that SOLVIT and GAWAIN are not included and that MAIN and EQNS are modified to include the initial condition "do loops" and constraints. The range of the do loops is such that the initial conditions are all within a reasonable approximation to an erect spin.

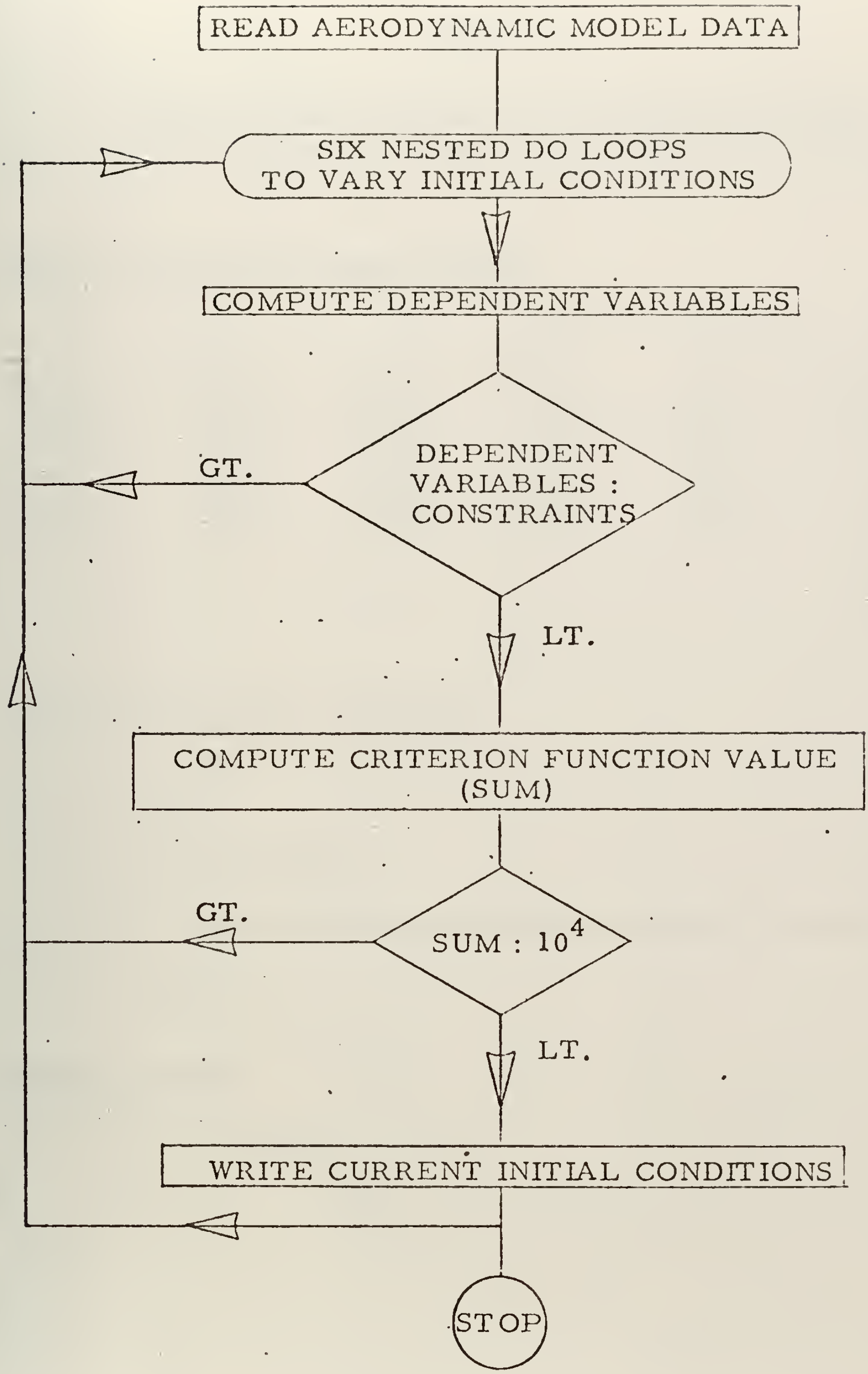


Figure C-1

Abbreviated Flowchart for the Criterion Function Search Computer Program

TABLE C-I

Pertinent Data Regarding the Criterion Function Search Program Independent and Dependent Variables

Initial Condition "Do Loop" Ranges

$\dot{Z}_o$	R	$\dot{\gamma}$	Θ_P	Ψ_R	Φ_Y
-150.0	25.0	10.0	-11.0	5.0	- 5.0
-200.0	100.0	20.0	-21.0	15.0	-15.0
-250.0	175.0	30.0	-31.0	25.0	-25.0
-300.0	250.0	40.0	-41.0	35.0	-35.0
-350.0	325.0	50.0	-51.0	45.0	-45.0
-400.0	400.0	60.0	-61.0	55.0	-55.0
					-65.0
					-75.0
					-85.0

Dependent Variable Constraints

$\beta = 0^\circ \pm 40^\circ$

$\alpha = -1.0^\circ$ to $+89.0^\circ$

$V_r \leq 589.0$ (corresponds to 300 knots equivalent airspeed)

$n \leq 2.0$ g's

Fixed Constants

$\rho = .001756$ slugs/ft^3

	F-111	F-4
$\delta_a =$	10.0	21.0
$\delta_r =$	-15.0	-30.0
$\delta_e =$	-25.0	-30.0

```
      IMPLICIT REAL*8(A-H,O-Z,$)                                          MAI00010
      EXTERNAL FIX                                                        MAI00020
      COMMON NFLAG                                                        MAI00030
      COMMON/WORKA/$MASS,RHO,CBAR,GRAV,AREA,$IX,$IY,$IZ,PI,IMAX,JMAX,     MAI00040
     1A(22),B(9),CFORCM(22,9,22),KEY(42),AILDEF,RUDDEF,ELEDEF
      COMMON/WORKB/ALPHA,BETA,R(12)
      REAL NAME(42) ,TYPE                                                 MAI00070
      INTEGER P(9),A1,A2                                                  MAI00080
      DIMENSION N1(42),N3(42),F(12)                                       MAI00090
      CALL ERRSET(207,100,-1,1,FIX,209)
      READ(5,1010) TYPE,IMAX,JMAX,(A(II),II=1,IMAX),(B(J),J=1,JMAX)       MAI00110
 1010 FORMAT(A4,2I3,7F10.2/(8F10.2))                                      MAI00120
      N1CNTR=0                                                            MAI00130
      N3CNTR=0                                                            MAI00140
      DO 121 K1=1,42                                                      MAI00150
      READ (5,1009)NA,NAME(NA),KEY(NA)                                    MAI00160
      WRITE(6,1009) NA,NAME(NA),KEY(NA)                                   MAI00170
      IF(KEY(NA).EQ.1)READ(5,1000)(CFORCM(I,1,NA),I=1,IMAX)               MAI00180
      IF(KEY(NA).EQ.2)READ(5,1000)((CFORCM(I,J,NA),J=1,JMAX),I=1,IMAX)    MAI00190
      IF(KEY(NA).EQ.0) GO TO 121                                          MAI00200
      IF(KEY(NA).EQ.2) GO TO 122                                          MAI00210
      N1CNTR=N1CNTR+1                                                     MAI00220
      N1(N1CNTR)=NA                                                       MAI00230
      GO TO 121                                                           MAI00240
  122 N3CNTR=N3CNTR+1                                                     MAI00250
      N3(N3CNTR)=NA                                                       MAI00260
  121 CONTINUE                                                            MAI00270
      N1MAX=N1CNTR                                                        MAI00280
      N3MAX=N3CNTR                                                        MAI00290
 1009 FORMAT(I10,10X,A4,10X,I6)                                           MAI00300
      READ(5,1000)$MASS,RHO,CBAR,GRAV,AREA,$IX,$IY,$IZ                    MAI00310
      READ(5,1000)AILDEF,RUDDEF,ELEDEF                                    MAI00320
 1000 FORMAT(8F10.6)                                                      MAI00330
      DO 123 K2=1,N3MAX                                                   MAI00340
      DO 150 JJ=1,JMAX                                                    MAI00350
  150 P(JJ)=IDINT(B(JJ))                                                  MAI00360
      I1=N3(K2)                                                           MAI00370
      WRITE(6,124 ) NAME(I1     ), (P(J),J=1,JMAX)                        MAI00380
      DO 125 K3=1,IMAX                                                    MAI00390
      A1=IDINT(A(K3))                                                     MAI00400
  125 WRITE(6,126 )A1          ,(CFORCM(K3,J,I1     ),J=1,JMAX)           MAI00410
  124 FORMAT('0',T55,'INPUT DATA'/////,T52,A4,'(ALPHA,BETA)'////          MAI00420
     1' ','BETA: ', 9(3X,I3,5X)///' ','ALPHA' )                           MAI00430
  126 FORMAT(' ',I3,10(3X,F8.4))                                          MAI00440
  123 CONTINUE                                                            MAI00450
      M1=1                                                                MAI00460
      M2=10                                                               MAI00470
      DO 127 K4=1,3                                                       MAI00480
```

```
      IF(N1MAX.LE.10)M2=N1MAX                                           MAI00490
      WRITE(6,130) (NAME(N1(I)),I=M1,M2)                                MAI00500
  130 FORMAT('0',T55,'INPUT DATA' //////                                MAI00510
     1' ','ALPHA',3X,10(A4,7X))                                         MAI00520
      DO 129 K6=1,IMAX                                                  MAI00530
      A2=IDINT(A(K6))                                                   MAI00540
  129 WRITE(6,126)A2          ,(CFORCM(K6,1, N1(I)),I=M1,M2)            MAI00550
      M1=M1+10                                                          MAI00560
      IF(M2.EQ.N1MAX)GO TO 128                                          MAI00570
      M2=M2+10                                                          MAI00580
      IF(N1MAX-M2)132,127,127                                           MAI00590
  132 M2=N1MAX                                                          MAI00600
  127 CONTINUE                                                          MAI00610
  128 PI=3.141593                                                       MAI00620
      DO 426 I=1,IMAX                                                   MAI00630
  426 A(I)=A(I)*PI/180.0                                                MAI00640
      DO 427 J=1,JMAX                                                   MAI00650
  427 B(J)=B(J)*PI/180.0                                                MAI00660
C     DO 555 K10=1,300                                                  MAI00670
C     READ(5,1003,END=450)(R(I),I=7,12),ICCODE                          MAI00680
C     GO TO 60                                                          MAI00690
   70 WRITE(6,100)                                                      MAI00700
  100 FORMAT('1',T49,'INITIAL INPUT AND COMPUTED DATA'/////             MAI00710
     1T53,'AIRCRAFT DATA'///' ',T8,'MASS',T22,'CHORD',T35,'WING AREA',T5MAI00720
     24,'IX',T69,'IY',T84,'IZ',T96,'GRAVITY',T111,'DENSITY'/            MAI00730
     3' ',T7,'(SLUGS)',T21,'(FEET)',T35,'(FEET**2)',T50,'**********  (SMAI00740
     4LUG*FEET**2)  **********',T94,'(FT/SEC**2)',T107,'(SLUGS/FT**3)') MAI00750
      WRITE(6,101)$MASS,CBAR,AREA,$IX,$IY,$IZ,GRAV,RHD                  MAI00760
  101 FORMAT('0',8(E13.4,2X)///)                                        MAI00770
      WRITE(6,8001)AILDEF,RUDDEF,ELEDEF                                 MAI00780
 8001 FORMAT('0',T50,'CONTROL DEFLECTIONS'/  ' ',T55,'(DEGREES)'///' ', MAI00790
     1T36,'AILERON',T57,'RUDDER',T76,'ELEVATOR'// ' ',T37,F6.2,T57,F6.2,MAI00800
     2T76,F6.2/////)                                                    MAI00810
      READ(5,7001)M1,M2,M3,M4,M5,M6,NAB,NG,NS,M7,NSOL,NERR
      WRITE(6,7002) M1,M2,M3,M4,M5,M6,NAB,NG,NS,NSOL,NERR
      DO 7000 I1=M1,6
      F(7)=-100.0-I1*50.0
      DO 7000 I2=M2,6
      F(8)=-50.0+I2*75.0
      DO 7000 I3=M3,6
      F(9)= 0.0 +I3*10.0
      DO 7000 I4= M4,6
      F(10)=9.0-I4*10.0
      DO 7000 I5=M5,6
      F(11)=-5.0 +I5*10.0
      DO 7000 I6 =M6,9
      F(12)=5.0-I6*10.0
      DO 138 I=7,12
```

```
138   R(I)=F(I)
      CALL EULER(ALF,BET,1)
      CALL EQNS (&1,&2,&3,&4)
      NSOL=NSOL+1
      GO TO 6999
 1    NAB=NAB +1
      GO TO 6999
2     NG=NG+1
      GO TO 6999
3     NS=NS+1
      GO TO 6999
4     NERR=NERR+1
6999  CALL EULER(ALF,BET,0)
      M7=M7+1
      IF(M7.EQ.50)WRITE(6,7002)I1,I2,I3,I4,I5,I6,NAB,NG,NS,NSOL,NERR
      IF(M7.EQ.50)M7=0
7000  CONTINUE
      WRITE(6,7002)I1,I2,I3,I4,I5,I6,NAB,NG,NS,NSOL,NERR
      STOP
7001  FORMAT(6I1,4X,6I10)
7002  FORMAT(' ',6I1,4X,5I10)
      END


      SUBROUTINE EQNS (*,*,*,*)
      IMPLICIT REAL*8(A-H,O-Z,$)                                        EQN00020
      EXTERNAL FIX                                                      EQN00030
      COMMON NFLAG                                                      EQN00040
      COMMON/WORKA/$MASS,RHO,CBAR,GRAV,AREA,$IX,$IY,$IZ,PI,IMAX,JMAX,   EQN00050
     1A(22),B(9),CFORCM(22,9,22),KEY(42),AILDEF,RUDDEF,ELEDEF
      COMMON/WORKB/ALPHA,BETA,R(12)                                     MAI00060
         DIMENSION  C(42)                                               EQN00080
C          COMPUTATION OF THE DIRECTION COSINES                         EQN00090
C                                                                       EQN00100
      NFLAG=1                                                           EQN00110
      A11=DCOS(R(12))*DCOS(R(11))-DSIN(R(12))*DSIN(R(11))*DCOS(R(10))   EQN00120
      A12=DCOS(R(12))*DSIN(R(11))+DSIN(R(12))*DCOS(R(11))*DCOS(R(10))   EQN00130
      A13=DSIN(R(10))*DSIN(R(12))                                       EQN00140
      A21=-DSIN(R(12))*DCOS(R(11))-DCOS(R(12))*DSIN(R(11))*DCOS(R(10))  EQN00150
      A22=-DSIN(R(12))*DSIN(R(11))+DCOS(R(12))*DCOS(R(11))*DCOS(R(10))  EQN00160
      A23=DSIN(R(10))*DCOS(R(12))                                       EQN00170
      A31=DSIN(R(10))*DSIN(R(11))                                       EQN00180
      A32=-DSIN(R(10))*DCOS(R(11))                                      EQN00190
      A33=DCOS(R(10))                                                   EQN00200
C                                                                       EQN00210
      ALPHA= DATAN((R(9)*R(8)*A32 + R(7)*A33)                           EQN00220
     1/(R(9)*R(8)*A12 + R(7)*A13))                                      EQN00230
      IF(NFLAG.EQ.0) RETURN 4
```

```
      BETA= DATAN((R(9)*R(8)*A22 + R(7)*A23)                            EQN00240
     1/(R(9)*R(8)*A12 + R(7)*A13 ))                                     EQN00250
      IF(NFLAG.EQ.0) RETURN 4
      IF((ALPHA.GT.A(IMAX)).OR.(ALPHA.LT.A(1)).OR.(BETA.GT.B(JMAX)).OR.
     1(BETA.LT.B(1)))RETURN 1
      IF(R(9)**2*R(8)/GRAV.GT.2.0)RETURN 2
   46 DO 47 NA=1,42                                                     EQN00260
      CALL INTER(NA,C(NA))                                              EQN00270
   47 CONTINUE                                                          EQN00280
      VEL=DSQRT(R(7)**2 + (R(9)*R(8))**2 )                              EQN00290
      IF(NFLAG.EQ.0) RETURN 4
      FAC13=CBAR*R(9 )*A13/2.0/VEL                                      EQN00300
      FAC23=CBAR*R(9 )*A23/2.0/VEL                                      EQN00310
      FAC33=CBAR*R(9 )*A33/2.0/VEL                                      EQN00320
      CFX=C(1) +C(25)*FAC13 + C(26)*FAC23 + C(23)*FAC33                 EQN00330
     1+C(27)*AILDEF + C(24)*RUDDEF + C( 7)*ELEDEF                       EQN00340
      CFY=C(2) +C(8 )*FAC13 + C(28)*FAC23 + C(9)*FAC33                  EQN00350
     1+C(10)*AILDEF + C(11)*RUDDEF + C(29)*ELEDEF                       EQN00360
      CFZ=C(3) +C(30)*FAC13 + C(31)*FAC23 + C(32)*FAC33                 EQN00370
     1+C(33)*AILDEF + C(34)*RUDDEF + C(12)*ELEDEF                       EQN00380
      CMX=C(4) + C(13)*FAC13+ C(35)*FAC23 + C(14)*FAC33                 EQN00390
     1+C(15)*AILDEF + C(16)*RUDDEF + C(36)*ELEDEF                       EQN00400
      CMY=C(5) +C(37)*FAC13 + C(17)*FAC23 + C(38)*FAC33                 EQN00410
     1+C(39)*AILDEF + C(40)*RUDDEF + C(18)*ELEDEF                       EQN00420
      CMZ=C(6) +C(19)*FAC13 + C(41)*FAC23 + C(20)*FAC33                 EQN00430
     1+C(21)*AILDEF + C(22)*RUDDEF + C(42)*ELEDEF                       EQN00440
   12 FAC1=((R(9)*R(8))**2 + R(7)**2 )                                  EQN00450
      IF(NFLAG.EQ.0) RETURN 4
      FAC2=RHO * AREA / 2.0/$MASS                                       EQN00460
      R(1)=-GRAV-(CFX*A13 +CFY*A23 + CFZ*A33)*FAC2*FAC1                 EQN00470
      IF(NFLAG.EQ.0) RETURN 4
      R(2)=R(9)**2*R(8)-(CFX*A11 + CFY*A21 + CFZ*A31)*FAC2*FAC1         EQN00480
      IF(NFLAG.EQ.0) RETURN 4
  202 R(3)=CFX*A12+CFY*A22+CFZ*A32                                      EQN00490
      IF(NFLAG.EQ.0) RETURN 4
      R(4)=2.0*($IX*(DSIN(R(12))**2)+$IY*(DCOS(R(12))**2)-$IZ)*R(9)**2  EQN00500
     1*DSIN(R(10))*DCOS(R(10))+FAC1*RHO*AREA*CBAR                       EQN00510
     2*(CMX*DCOS(R(12))-CMY*DSIN(R(12)))                                EQN00520
      IF(NFLAG.EQ.0) RETURN 4
      R(5)=CMX*A13+CMY*A23+CMZ*A33                                      EQN00530
      IF(NFLAG.EQ.0) RETURN 4
      R(6)=2.0*($IX-$IY)*R(9)**2*(DSIN(R(10))**2)*DSIN(R(12))*DCOS(R(12)EQN00540
     1)+RHO*AREA*CBAR*FAC1*CMZ                                          EQN00550
      IF(NFLAG.EQ.0) RETURN 4
      SUM1=DABS(R(3))+DABS(R(4))+DABS(R(5))+DABS(R(6))                  EQN00560
  400 SUM=SUM1 +DABS(R(1)) +DABS(R(2))                                  EQN00580
      IF(SUM.GT.10.0**4) RETURN 3
      CALL EULER (ALF,BET,0)
```

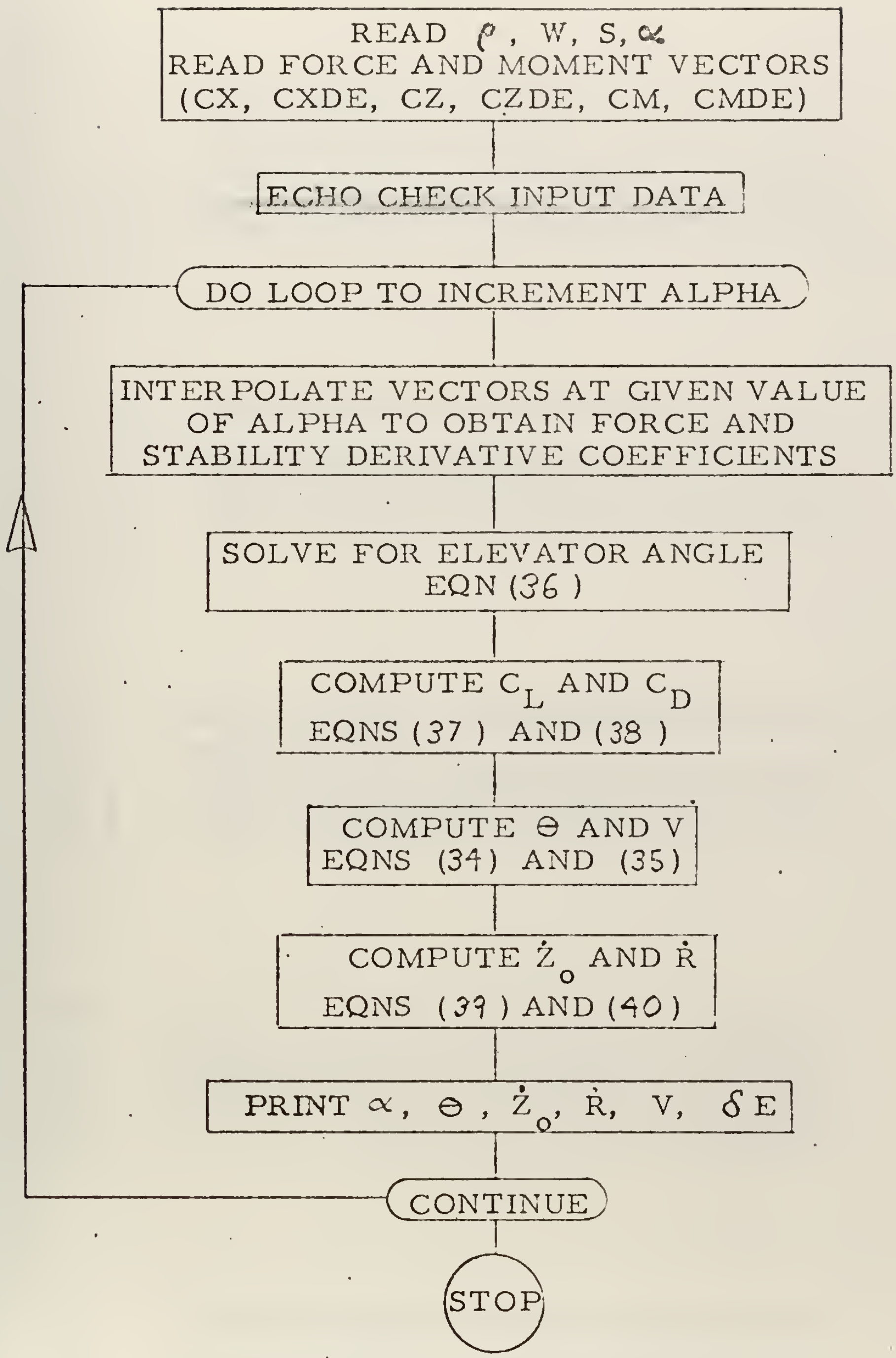
```
      WRITE(6,7003)(R(I),I=1,12),SUM
 7003 FORMAT(' ',13(E9.3,1X))
      WRITE(6,7004) ALF,BET
 7004 FORMAT(' ',2F10.2)
      CALL EULER(ALF,BET,1)
  300 RETURN                                                            EQN00600
      END                                                               EQN00610


      SUBROUTINE EULER(ALF,BET,N)                                       EUL00220
      IMPLICIT REAL *8 (A-H,O-Z,$)                                      EUL00230
      COMMON/WORKA/$MASS,RHO,CBAR,GRAV,AREA,$IX,$IY,$IZ,PI,IMAX,JMAX,
     1A(22),B(9),CFORCM(22,9,22),KEY(42),AILDEF,RUDDEF,ELEDEF           MAI00020
      COMMON/WORKB/ALPHA,BETA,R(12)
      PI=3.141593                                                       EUL00260
C                                                                       EUL00270
C         N=0:  CONVERT REFERENCE EULER ANGLES TO ORDERED EULER ANGLES  EUL00280
C               AND THEN CONVERT ANGULAR VARIABLES FROM RADIANS TO      EUL00290
C               DEGREES                                                 EUL00300
C                                                                       EUL00310
C         N=1:  CONVERT ANGULAR VARIABLES FROM DEGREES TO RADIANS AND   EUL00320
C               THEN CONVERT ORDERED EULER ANGLES TO REFERENCE EULER    EUL00330
C               ANGLES                                                  EUL00340
C                                                                       EUL00350
      IF(N.EQ.1)GO TO 2                                                 EUL00360
C                                                                       EUL00370
C         EULER ANGLE CONVERSION: REFERENCE TO ORDERED                  EUL00380
C                                                                       EUL00390
      X=R(10)                                                           EUL00400
      Y=R(11)                                                           EUL00410
      Z=R(12)                                                           EUL00420
      CTERM=DATAN(DTAN(Y/2.0)*DSIN((PI/2.0-X)/2.0)/DSIN((PI/2.0+X)/2.0))EUL00430
      DTERM=DATAN(DTAN(Y/2.0)*DCOS((PI/2.0-X)/2.0)/DCOS((PI/2.0+X)/2.0))EUL00440
      PSI=Z+CTERM+DTERM                                                 EUL00450
      THETA=DTERM-CTERM                                                 EUL00460
      PHI=PI/2.0-2.0*DATAN(DSIN(CTERM)/(DSIN(DTERM)*DTAN((PI/2.0-X)/2.0)EUL00470
     1))                                                                EUL00480
      R(10)=THETA*130.0/PI                                              EUL00490
      R(11)=PSI*180.0/PI                                                EUL00500
      R(12)=PHI*180.0/PI                                                EUL00510
      R(9)=R(9)*180.0/PI                                                EUL00520
      ALF=ALPHA*180.0/PI                                                EUL00530
      BET=BETA*180.0/PI                                                 EUL00540
      GO TO 3                                                           EUL00550
C                                                                       EUL00560
C         EULER ANGLE CONVERSION: ORDERED TO REFERENCE                  EUL00570
C                                                                       EUL00580
    2 DO 4 I=9,12                                                       EUL00590
```

```
 4    R(I)=R(I)*PI/180.0                                        EUL00600
      X=R(10)                                                   EUL00610
      Y=R(11)                                                   EUL00620
      Z=R(12)                                                   EUL00630
      ATERM=DATAN(DTAN(X/2.0)*DTAN(Z/2.0))                      EUL00640
      BTERM=DATAN(DTAN(X/2.0)/DTAN(Z/2.0))                      EUL00650
      PHI=Y-ATERM-BTERM                                         EUL00660
      PSI=BTERM-ATERM                                           EUL00670
      THETA=2.0*DATAN(DSIN(ATERM)/(DSIN(BTERM)*DTAN(Z/2.0)))    EUL00680
      R(10)=THETA                                               EUL00690
      R(11)=PSI                                                 EUL00700
      R(12)=PHI                                                 EUL00710
 3    RETURN                                                    EUL00720
      END                                                       EUL00730


      SUBROUTINE INTER(NA,VALUE)                                INT00010
      IMPLICIT REAL *8 (A-H,O-Z,$)
      COMMON/WORKA/$MASS,RHO,CBAR,GRAV,AREA,$IX,$IY,$IZ,PI,IMAX,JMAX,
     1A(22),B(9),CFORCM(22,9,22),KEY(42),AILDEF,RUDDEF,ELEDEF
      COMMON/WORKB/ALPHA,BETA,R(12)
      DIMENSION AMAT(22,9),YY(9),YI(22)
      II=KEY(NA)+1                                              INT00080
C
C         II=1: UNDEFINED STABILITY DERIVATIVE
C         II=2: COEFFICIENT VECTOR
C         II=3: COEFFICIENT MATRIX
C
      GO TO (1,2,4 ),II
C
C         UNDEFINED STABILITY DERIVATIVES ARE EQUATED TO ZERO
C
 1    VALUE=0.0                                                 INT00100
      RETURN                                                    INT00110
C
C         VECTOR INTERPOLATION
C
 2    DO 3 I=1,IMAX                                             INT00120
 3    YI(I)=(CFORCM(I,1,NA))
      CALL SPLIN1(A,YI,IMAX,ALPHA,VALUE)
      RETURN                                                    INT00150
C
C         MATRIX INTERPOLATION
C
 4    DO 5 I=1,IMAX                                             INT00160
      DO 5 J= 1,JMAX                                            INT00170
 5    AMAT(I,J)=(CFORCM(I,J,NA))
      DO 7 NR=1,IMAX                                            INT00190
```

```
      DO 6 NC=1,JMAX                                                    INT00200
    6 YY(NC)=AMAT(NR,NC)
C
C        USE SPLIN1 TO INTERPOLATE/FILL UTILITY VECTOR (YI) WHICH
C        CORRESPONDS TO B=BETA FOR ALL VALUES OF A
C
    7 CALL SPLIN1(B,YY,JMAX,BETA,YI(NR))
C        ASSIGN VALUES
C        NOW INTERPOLATE YI AT A=ALPHA
C
      CALL SPLIN1(A,YI,IMAX,ALPHA,VALUE)                                INT00230
      RETURN                                                            INT00240
      END


      SUBROUTINE FIX(IRETCD,IERNO)
      IMPLICIT REAL*8(A-H,O-Z,$)
      COMMON NFLAG
      NFLAG=0
      IRETCD=0
      RETURN
      END
```

APPENDIX D

TRIM PROGRAM



Subroutines required: SPLIN1 and SPLICO

DENSITY	WEIGHT	AREA			
.15530E-02	.50024E 05	.52500E 03			

ALPHA	THETA	SINKRATE	RADIUS RATE	VELOCITY	ELEVATOR ANGLE
0.1000E 01	-.6122E 02	0.1865E 04	0.9824E 03	0.2108E 04	0.1453E 01
0.2000E 01	-.1813E 02	0.3989E 03	0.1088E 04	0.1159E 04	0.8292E 00
0.3000E 01	-.1125E 02	0.2135E 03	0.8410E 03	0.8676E 03	0.1862E 00
0.4000E 01	-.8302E 01	0.1543E 03	0.7075E 03	0.7241E 03	-.4954E 00
0.5000E 01	-.6411E 01	0.1258E 03	0.6231E 03	0.6357E 03	-.1233E 01
0.6000E 01	-.4890E 01	0.1084E 03	0.5636E 03	0.5740E 03	-.2032E 01
0.7000E 01	-.3578E 01	0.9673E 02	0.5180E 03	0.5270E 03	-.2846E 01
0.8000E 01	-.2433E 01	0.8858E 02	0.4810E 03	0.4891E 03	-.3622E 01
0.9000E 01	-.1449E 01	0.8300E 02	0.4501E 03	0.4576E 03	-.4308E 01
0.1000E 02	-.6230E 00	0.7945E 02	0.4236E 03	0.4310E 03	-.4852E 01
0.1100E 02	0.4709E-01	0.7756E 02	0.4008E 03	0.4082E 03	-.5234E 01
0.1200E 02	0.5763E 00	0.7701E 02	0.3811E 03	0.3888E 03	-.5534E 01
0.1300E 02	0.9836E 00	0.7750E 02	0.3641E 03	0.3723E 03	-.5857E 01
0.1400E 02	0.1288E 01	0.7882E 02	0.3494E 03	0.3582E 03	-.6308E 01
0.1500E 02	0.1509E 01	0.8081E 02	0.3368E 03	0.3464E 03	-.6987E 01
0.1600E 02	0.1671E 01	0.8326E 02	0.3260E 03	0.3364E 03	-.7929E 01
0.1700E 02	0.1823E 01	0.8575E 02	0.3161E 03	0.3275E 03	-.8893E 01
0.1800E 02	0.1995E 01	0.8791E 02	0.3065E 03	0.3188E 03	-.9572E 01
0.1900E 02	0.2200E 01	0.8950E 02	0.2964E 03	0.3097E 03	-.9663E 01
0.2000E 02	0.2431E 01	0.9041E 02	0.2855E 03	0.2995E 03	-.8875E 01
0.2100E 02	0.2661E 01	0.9074E 02	0.2737E 03	0.2884E 03	-.7080E 01
0.2200E 02	0.2842E 01	0.9102E 02	0.2620E 03	0.2773E 03	-.4733E 01
0.2300E 02	0.2950E 01	0.9166E 02	0.2512E 03	0.2674E 03	-.2416E 01
0.2400E 02	0.2984E 01	0.9289E 02	0.2418E 03	0.2590E 03	-.6701E 00
0.2500E 02	0.2955E 01	0.9483E 02	0.2342E 03	0.2526E 03	0.5426E-05
0.2600E 02	0.2883E 01	0.9751E 02	0.2284E 03	0.2484E 03	-.7209E 00
0.2700E 02	0.2800E 01	0.1007E 03	0.2242E 03	0.2458E 03	-.2548E 01
0.2800E 02	0.2726E 01	0.1043E 03	0.2210E 03	0.2444E 03	-.5081E 01
0.2900E 02	0.2671E 01	0.1081E 03	0.2184E 03	0.2437E 03	-.7954E 01
0.3000E 02	0.2636E 01	0.1119E 03	0.2162E 03	0.2434E 03	-.1084E 02
0.3100E 02	0.2612E 01	0.1156E 03	0.2138E 03	0.2430E 03	-.1342E 02
0.3200E 02	0.2582E 01	0.1189E 03	0.2109E 03	0.2421E 03	-.1542E 02
0.3300E 02	0.2523E 01	0.1218E 03	0.2070E 03	0.2402E 03	-.1652E 02
0.3400E 02	0.2409E 01	0.1241E 03	0.2018E 03	0.2368E 03	-.1641E 02
0.3500E 02	0.2220E 01	0.1255E 03	0.1949E 03	0.2318E 03	-.1471E 02

```
      REAL*8 CX(10),CXDE(10),CZ(10),CZDE(10),CM(10),CMDE(10),A(10),
     1ALPH,CMT,CMDET,CXTD,CXDETD,CZTD,CZDETD
      DIMENSION X(2)
      PI=3.141592
      READ(5,1000)RHO,W,S
      READ(5,1000)CX
      READ(5,1000)CXDE
      READ(5,1000)CZ
      READ(5,1000)CZDE
      READ(5,1000)CM
      READ(5,1000)CMDE
      READ(5,1000)A
      WRITE(6,2000)CX,CXDE,CZ,CZDE,CM,CMDE,A
      DO 10 I=1,10
10    A(I)=A(I)*PI/180.0
      WRITE(6,2001) RHO,W,S
2001  FORMAT('0',T10,'DENSITY',T30,'WEIGHT',T50,'AREA'/' ',3(10X,E10.5))
      WRITE(6,2003)
2003  FORMAT('0',T10,'ALPHA',T30,'THETA',T50,'SINKRATE',T70,'RADIUS RATE
     1',T90,'VELOCITY',T110,'ELEVATOR ANGLE'///)
      READ (5,1000) ALPHA
      DO 20 J=1,55
      ALPHA=ALPHA+1.0
      IF(ALPHA.GT.35.0) STOP
      ALPHA=ALPHA*PI/180.0
      ALPH =DBLE(ALPHA)
      CALL SPLIN1(A,CM,10,ALPH ,CMT)
      CALL SPLIN1(A,CMDE,10,ALPH ,CMDET)
      CALL SPLIN1(A,CX,10,ALPH ,CXTD)
      CALL SPLIN1(A,CXDE,10,ALPH ,CXDETD)
      CALL SPLIN1(A,CZ,10,ALPH ,CZTD)
      CALL SPLIN1(A,CZDE,10,ALPH ,CZDETD)
      DET=SNGL(-CMT/CMDET)
      CXT=SNGL(CXTD)
      CXDET=SNGL(CXDETD)
      CZT=SNGL(CZTD)
      CZDET=SNGL(CZDETD)
      CLT=-(CZT+CZDET*DET)*COS(ALPHA) +(CXT+CXDET*DET)*SIN(ALPHA)
      CDT=-(CXT+CXDET*DET)*COS(ALPHA)-(CZT+CZDET*DET)*SIN(ALPHA)
      X(1)=ATAN(CDT/CLT)
      X(2)=SQRT(2.0*W/(RHO*S*(CLT*COS(X(1))+CDT*SIN(X(1)))) )
      THETA=(ALPHA-X(1))*180.0/PI
      ZODOT=X(2)*SIN(X(1))
      RDOT=X(2)*COS(X(1))
      ALPHA=ALPHA*180.0/PI
      WRITE(6,2002)ALPHA,THETA,ZODOT,RDOT,X(2),DET
2002  FORMAT(' ',6(10X,E10.4))
20    CONTINUE
```

```
2000 FORMAT('0',10E13.5)
1000 FORMAT(8F10.6)
     STOP
     END
```

APPENDIX E

AERODYNAMIC FORCE AND MOMENT MODELS

I. F-111 Data

A.

Mass	1555.0	slugs
Chord	9.04	feet
Wing Area	525	$feet^2$
I_x	50,000	slug ft^2
I_y	315,200	slug ft^2
I_z	351,500	slug ft^2

B. Attached data F-111 reduced to 6 x 9

II. F-4 Data

A.

Mass	1345	slugs
Chord	16.03	feet
Wing Area	530	$feet^2$
I_x	23,000	slug ft^2
I_y	134,000	slug ft^2
I_z	152,000	slug ft^2

B. Attached data (F-4) reduced to 6 x 9.

F-4

CNDR(ALPHA,BETA)

ALPHA \ BETA:	-40	-30	-20	-10	0	10	20	30	40
-1	-0.000590	-0.000530	-0.000950	-0.001130	-0.001160	-0.001130	-0.000950	-0.000530	-0.000590
4	-0.000580	-0.000530	-0.001100	-0.001130	-0.001160	-0.001130	-0.001100	-0.000530	-0.000580
9	-0.000590	-0.000530	-0.001040	-0.001130	-0.001130	-0.001130	-0.001040	-0.000530	-0.000590
14	-0.000580	-0.000520	-0.001300	-0.001120	-0.001050	-0.001120	-0.001300	-0.000520	-0.000580
19	-0.000560	-0.000510	-0.000880	-0.000730	-0.000910	-0.000730	-0.000880	-0.000510	-0.000560
24	-0.000460	-0.000460	-0.000570	-0.000450	-0.000750	-0.000450	-0.000570	-0.000460	-0.000460
29	-0.000440	-0.000450	-0.000400	-0.000280	-0.000520	-0.000280	-0.000400	-0.000450	-0.000440
34	-0.000420	-0.000430	-0.000270	-0.000210	-0.000360	-0.000210	-0.000270	-0.000430	-0.000420
39	-0.000420	-0.000390	-0.000220	-0.000170	-0.000220	-0.000170	-0.000220	-0.000390	-0.000420
44	-0.000380	-0.000220	-0.000140	-0.000150	-0.000040	-0.000150	-0.000140	-0.000220	-0.000380
49	-0.000290	-0.000150	-0.000150	-0.000120	0.0	-0.000120	-0.000150	-0.000150	-0.000290
54	-0.000200	-0.000140	-0.000160	-0.000110	0.0	-0.000110	-0.000160	-0.000140	-0.000200
59	-0.000070	-0.000150	-0.000130	-0.000090	0.0	-0.000090	-0.000130	-0.000150	-0.000070
64	-0.000050	-0.000130	-0.000120	0.000085	0.0	0.000085	-0.000120	-0.000130	-0.000050
69	-0.000110	-0.000090	-0.000130	-0.000080	0.0	-0.000080	-0.000130	-0.000090	-0.000110
74	-0.000160	-0.000070	-0.000130	0.000070	0.0	0.000070	-0.000130	-0.000070	-0.000160
79	-0.000180	-0.000100	-0.000140	-0.000080	0.0	-0.000080	-0.000140	-0.000100	-0.000180
84	-0.000190	-0.000150	-0.000160	0.000080	0.0	0.000080	-0.000150	-0.000150	-0.000190
89	-0.000210	-0.000190	-0.000180	-0.000100	0.0	-0.000100	-0.000180	-0.000190	-0.000210

F-4

CNDA(ALPHA,BETA)

ALPHA \ BETA:	-40	-30	-20	-10	0	10	20	30	40
-1	0.000180	0.000050	0.000070	-0.000020	0.0	-0.000020	0.000070	0.000050	0.000180
4	0.000110	0.000060	0.000070	-0.000010	0.0	-0.000010	0.000070	0.000060	0.000110
9	0.000110	0.000070	0.000080	0.000030	0.000010	0.000030	0.000080	0.000070	0.000110
14	0.000080	0.000090	0.000080	0.000070	0.000070	0.000070	0.000080	0.000090	0.000080
19	0.000050	0.000070	0.000120	0.000120	0.000160	0.000120	0.000120	0.000070	0.000050
24	0.000010	0.000030	0.000130	0.000150	0.000150	0.000150	0.000130	0.000030	0.000010
29	-0.000030	0.0	0.000130	0.000160	0.000150	0.000160	0.000130	0.0	-0.000030
34	-0.000020	-0.000010	0.000130	0.000160	0.000130	0.000160	0.000130	-0.000010	-0.000020
39	-0.000030	0.0	0.000140	0.000170	0.000200	0.000170	0.000140	0.0	-0.000030
44	0.000010	0.000020	0.000140	0.000160	0.000170	0.000160	0.000140	0.000020	0.000010
49	0.000020	0.000030	0.000150	0.000140	0.000200	0.000140	0.000150	0.000030	0.000020
54	0.000040	0.000040	0.000160	0.000110	0.000170	0.000110	0.000160	0.000040	0.000040
59	0.000050	0.000060	0.000160	0.000130	0.000170	0.000130	0.000160	0.000060	0.000050
64	0.000060	0.000080	0.000160	0.000190	0.000200	0.000190	0.000160	0.000080	0.000060
69	0.000080	0.000100	0.000160	0.000240	0.000160	0.000240	0.000160	0.000100	0.000080
74	0.000100	0.000120	0.000160	0.000260	0.000130	0.000260	0.000160	0.000120	0.000100
79	0.000130	0.000150	0.000160	0.000270	0.000150	0.000270	0.000160	0.000150	0.000130
84	0.000150	0.000170	0.000150	0.000250	0.000170	0.000250	0.000150	0.000170	0.000150
89	0.000160	0.000180	0.000140	0.000230	0.000150	0.000230	0.000140	0.000180	0.000160

F-4

CN (ALPHA,BETA)

BETA: ALPHA	-40	-30	-20	-10	0	10	20	30	40
-1	-0.238800	-0.202600	-0.149500	-0.058610	0.0	0.058610	0.149500	0.202600	0.238800
4	-0.242400	-0.209800	-0.152000	-0.062710	0.0	0.062710	0.152000	0.209800	0.242400
9	-0.242400	-0.211100	-0.149500	-0.053550	0.0	0.053550	0.149500	0.211100	0.242400
14	-0.241200	-0.207400	-0.141600	-0.050650	0.0	0.050650	0.141600	0.207400	0.241200
19	-0.123000	-0.097690	-0.078390	-0.015680	0.0	0.015680	0.078390	0.097690	0.123000
24	-0.061510	-0.001206	0.030150	0.065130	0.0	-0.065130	-0.030150	0.001206	0.061510
29	-0.053070	0.033770	0.112900	0.090450	0.0	-0.090450	-0.112900	-0.033770	0.053070
34	-0.045830	0.067540	0.099380	0.094070	0.0	-0.094070	-0.099380	-0.067540	0.045830
39	-0.018090	0.078390	0.103700	0.052580	0.0	-0.052580	-0.103700	-0.078390	0.018090
44	-0.008442	0.086830	0.080800	0.047030	0.0	-0.047030	-0.080800	-0.086830	0.008442
49	-0.033770	0.089250	0.033770	0.060300	0.0	-0.060300	-0.033770	-0.089250	0.033770
54	-0.007236	0.089250	0.115800	0.106100	0.0	-0.106100	-0.115800	-0.089250	0.007236
59	-0.006030	0.060300	0.094070	0.089250	0.0	-0.089250	-0.094070	-0.060300	0.006030
64	-0.008442	0.038590	0.107300	0.125400	0.0	-0.125400	-0.107300	-0.038590	0.008442
69	-0.031360	0.028940	0.068740	0.067540	0.0	-0.067540	-0.068740	-0.028940	0.031360
74	-0.079600	0.014470	0.042210	0.048240	0.0	-0.048240	-0.042210	-0.014470	0.079600
79	-0.095280	0.0	0.037390	0.039800	0.0	-0.039800	-0.037390	0.0	0.095280
84	-0.144700	-0.048240	0.003618	0.012060	0.0	-0.012060	-0.003618	0.048240	0.144700
89	-0.173700	-0.084420	-0.026530	0.006030	0.0	-0.006030	0.026530	0.084420	0.173700

F-4

CMDS(ALPHA,BETA)

BETA: ALPHA	-40	-30	-20	-10	0	10	20	30	40
-1	-0.007200	-0.009800	-0.009000	-0.009800	-0.010200	-0.009800	-0.009000	-0.009800	-0.007200
4	-0.007800	-0.010200	-0.009700	-0.010300	-0.009900	-0.010300	-0.009700	-0.010200	-0.007800
9	-0.007900	-0.010200	-0.010200	-0.010200	-0.010000	-0.010200	-0.010200	-0.010200	-0.007900
14	-0.007400	-0.009800	-0.010300	-0.009800	-0.010100	-0.009800	-0.010300	-0.009800	-0.007400
19	-0.006500	-0.009000	-0.010000	-0.009000	-0.009600	-0.009000	-0.010000	-0.009000	-0.006500
24	-0.005800	-0.007700	-0.008300	-0.007600	-0.008700	-0.007600	-0.008300	-0.007700	-0.005800
29	-0.005300	-0.006000	-0.006600	-0.006100	-0.007000	-0.006100	-0.006600	-0.006000	-0.005300
34	-0.005000	-0.004200	-0.005200	-0.005200	-0.004900	-0.005200	-0.005200	-0.004200	-0.005000
39	-0.004700	-0.003400	-0.004400	-0.004400	-0.003000	-0.004400	-0.004400	-0.003400	-0.004700
44	-0.004200	-0.003200	-0.004000	-0.004000	-0.001900	-0.004000	-0.004000	-0.003200	-0.004200
49	-0.003600	-0.003400	-0.004400	-0.003700	-0.001600	-0.003700	-0.004400	-0.003400	-0.003600
54	-0.003000	-0.004400	-0.004900	-0.003600	-0.002000	-0.003600	-0.004900	-0.004400	-0.003000
59	-0.002400	-0.006000	-0.005000	-0.003600	-0.002700	-0.003600	-0.005000	-0.006000	-0.002400
64	-0.003300	-0.005200	-0.004800	-0.004000	-0.003200	-0.004000	-0.004600	-0.005200	-0.003300
69	-0.004000	-0.004000	-0.004500	-0.004300	-0.003500	-0.004300	-0.004500	-0.004000	-0.004000
74	-0.004100	-0.003300	-0.004200	-0.004500	-0.003800	-0.004500	-0.004200	-0.003300	-0.004100
79	-0.004200	-0.003600	-0.003800	-0.003600	-0.004200	-0.003600	-0.003800	-0.003600	-0.004200
84	-0.004500	-0.003700	-0.003100	-0.002700	-0.004600	-0.002700	-0.003100	-0.003700	-0.004500
89	-0.005400	-0.003300	-0.001700	-0.002600	-0.003600	-0.002600	-0.001700	-0.003300	-0.005400

F-4

CZ (ALPHA,BETA)

BETA: ALPHA	-40	-30	-20	-10	0	10	20	30	40
-1	0.250000	0.075000	-0.120000	-0.120000	-0.120000	-0.120000	-0.120000	0.075000	0.250000
4	-0.050000	-0.210000	-0.370000	-0.370000	-0.370000	-0.370000	-0.370000	-0.210000	-0.050000
9	-0.330000	0.480000	-0.630000	-0.630000	-0.630000	-0.630000	-0.630000	0.480000	-0.330000
14	-0.570000	-0.720000	-0.860000	-0.860000	-0.860000	-0.860000	-0.860000	-0.720000	-0.570000
19	-0.780000	-0.940000	-1.330000	-1.330000	-1.330000	-1.330000	-1.330000	-0.940000	-0.780000
24	-0.940000	-1.060000	-1.190000	-1.190000	-1.190000	-1.190000	-1.190000	-1.060000	-0.940000
29	-1.030000	-1.150000	-1.260000	-1.260000	-1.260000	-1.260000	-1.260000	-1.150000	-1.030000
34	-1.120000	-1.220000	-1.320000	-1.320000	-1.320000	-1.320000	-1.320000	-1.220000	-1.120000
39	-1.190000	-1.290000	-1.390000	-1.390000	-1.390000	-1.390000	-1.390000	-1.290000	-1.190000
44	-1.250000	-1.350000	-1.460000	-1.460000	-1.460000	-1.460000	-1.460000	-1.350000	-1.250000
49	-1.330000	-1.430000	-1.530000	-1.530000	-1.530000	-1.530000	-1.530000	-1.430000	-1.330000
54	-1.400000	-1.500000	-1.600000	-1.600000	-1.600000	-1.600000	-1.600000	-1.500000	-1.400000
59	-1.460000	-1.550000	-1.640000	-1.640000	-1.640000	-1.640000	-1.640000	-1.550000	-1.460000
64	-1.510000	-1.590000	-1.670000	-1.670000	-1.670000	-1.670000	-1.670000	-1.590000	-1.510000
69	-1.550000	-1.620000	-1.690000	-1.690000	-1.690000	-1.690000	-1.690000	-1.620000	-1.550000
74	-1.580000	-1.640000	-1.700000	-1.700000	-1.700000	-1.700000	-1.700000	-1.640000	-1.580000
79	-1.570000	-1.640000	-1.700000	-1.700000	-1.700000	-1.700000	-1.700000	-1.640000	-1.570000
84	-1.540000	-1.640000	-1.700000	-1.700000	-1.700000	-1.700000	-1.700000	-1.640000	-1.540000
89	-1.560000	-1.620000	-1.680000	-1.680000	-1.680000	-1.680000	-1.680000	-1.620000	-1.560000

F-4

CM (ALPHA,BETA)

BETA: ALPHA	-40	-30	-20	-10	0	10	20	30	40
-1	0.100000	0.035000	-0.030000	-0.015000	0.0	-0.015000	-0.030000	0.035000	0.100000
4	0.095000	0.030000	-0.035000	-0.020000	-0.005000	-0.020000	-0.035000	0.030000	0.095000
9	0.090000	0.025000	-0.040000	-0.025000	-0.010000	-0.025000	-0.040000	0.025000	0.090000
14	0.075000	0.011000	-0.053000	-0.039000	-0.025000	-0.039000	-0.053000	0.011000	0.075000
19	0.075000	0.010000	-0.055000	-0.040000	-0.025000	-0.040000	-0.055000	0.010000	0.075000
24	0.051000	-0.014000	-0.079000	-0.065000	-0.050000	-0.065000	-0.079000	-0.014000	0.051000
29	0.032000	-0.034000	-0.100000	-0.086000	-0.072000	-0.086000	-0.100000	-0.034000	0.032000
34	0.016000	-0.049000	-0.114000	-0.100000	-0.085000	-0.100000	-0.114000	-0.049000	0.016000
39	0.010000	-0.055000	-0.121000	-0.106000	-0.091000	-0.106000	-0.121000	-0.055000	0.010000
44	0.005000	-0.070000	-0.135000	-0.120000	-0.105000	-0.120000	-0.135000	-0.070000	0.005000
49	-0.032000	-0.097000	-0.161000	-0.146000	-0.131000	-0.146000	-0.161000	-0.097000	-0.032000
54	-0.062000	-0.127000	-0.191000	-0.176000	-0.161000	-0.176000	-0.191000	-0.127000	-0.062000
59	-0.090000	-0.155000	-0.220000	-0.205000	-0.190000	-0.205000	-0.220000	-0.155000	-0.090000
64	-0.120000	-0.185000	-0.250000	-0.235000	-0.220000	-0.235000	-0.250000	-0.185000	-0.120000
69	-0.154000	-0.219000	-0.285000	-0.270000	-0.255000	-0.270000	-0.285000	-0.219000	-0.154000
74	-0.194000	-0.259000	-0.325000	-0.310000	-0.295000	-0.310000	-0.325000	-0.259000	-0.194000
79	-0.225000	-0.291000	-0.357000	-0.341000	-0.325000	-0.341000	-0.357000	-0.291000	-0.225000
84	-0.260000	-0.325000	-0.390000	-0.375000	-0.360000	-0.375000	-0.390000	-0.325000	-0.260000
89	-0.275000	-0.340000	-0.404000	-0.390000	-0.375000	-0.390000	-0.404000	-0.340000	-0.275000

F-4

CY (ALPHA,BETA)

BETA: ALPHA	-40	-30	-20	-10	0	10	20	30	40
-1	0.575000	0.457000	0.300000	0.129000	0.0	-0.129000	-0.300000	-0.457000	-0.575000
4	0.565000	0.450000	0.295000	0.125000	0.0	-0.125000	-0.295000	-0.450000	-0.565000
9	0.560000	0.450000	0.300000	0.125000	0.0	-0.125000	-0.300000	-0.450000	-0.560000
14	0.555000	0.430000	0.290000	0.120000	0.0	-0.120000	-0.290000	-0.430000	-0.555000
19	0.555000	0.408000	0.285000	0.110000	0.0	-0.110000	-0.285000	-0.408000	-0.555000
24	0.485000	0.335000	0.205000	0.053000	0.0	-0.053000	-0.205000	-0.335000	-0.485000
29	0.445000	0.335000	0.180000	0.050000	0.0	-0.050000	-0.180000	-0.335000	-0.445000
34	0.420000	0.295000	0.135000	0.065000	0.0	-0.065000	-0.135000	-0.295000	-0.420000
39	0.390000	0.235000	0.120000	0.083000	0.0	-0.083000	-0.120000	-0.235000	-0.390000
44	0.375000	0.208000	0.135000	0.105000	0.0	-0.105000	-0.135000	-0.208000	-0.375000
49	0.300000	0.145000	0.140000	0.065000	0.0	-0.065000	-0.140000	-0.145000	-0.300000
54	0.245000	0.125000	0.088000	0.076000	0.0	-0.076000	-0.088000	-0.125000	-0.245000
59	0.205000	0.070000	0.020000	0.090000	0.0	-0.090000	-0.020000	-0.070000	-0.205000
64	0.170000	0.050000	-0.010000	0.040000	0.0	-0.040000	0.010000	-0.050000	-0.170000
69	0.185000	0.065000	0.020000	0.035000	0.0	-0.035000	-0.020000	-0.065000	-0.185000
74	0.185000	0.090000	0.025000	0.010000	0.0	-0.010000	-0.025000	-0.090000	-0.185000
79	0.195000	0.095000	0.020000	-0.030000	0.0	0.030000	-0.020000	-0.095000	-0.195000
84	0.225000	0.130000	0.025000	-0.015000	0.0	0.015000	-0.025000	-0.130000	-0.225000
89	0.235000	0.150000	0.050000	-0.010000	0.0	0.010000	-0.050000	-0.150000	-0.235000

F-4

CL (ALPHA,BETA)

BETA: ALPHA	-40	-30	-20	-10	0	10	20	30	40
-1	-0.028940	-0.029430	-0.031360	-0.024120	0.0	-0.024120	-0.031360	-0.029430	-0.028940
4	-0.091660	-0.084420	-0.077190	-0.060300	0.0	-0.060300	-0.077190	-0.084420	-0.091660
9	-0.082010	-0.089250	-0.086830	-0.057890	0.0	-0.057890	-0.086830	-0.089250	-0.082010
14	-0.084420	-0.077190	-0.067540	-0.043420	0.0	-0.043420	-0.067540	-0.077190	-0.084420
19	-0.089250	-0.067540	-0.055480	-0.012060	0.0	-0.012060	-0.055480	-0.067540	-0.089250
24	-0.098890	-0.079600	-0.033770	-0.007236	0.0	-0.007236	-0.033770	-0.079600	-0.098890
29	-0.166400	-0.094070	-0.028940	-0.009648	0.0	-0.009648	-0.028940	-0.094070	-0.166400
34	-0.214700	-0.118200	-0.074770	-0.004824	0.0	-0.004824	-0.074770	-0.118200	-0.214700
39	-0.224300	-0.139900	-0.086830	-0.041000	0.0	-0.041000	-0.086830	-0.139900	-0.224300
44	-0.234000	-0.152000	-0.108500	-0.048240	0.0	-0.048240	-0.108500	-0.152000	-0.234000
49	-0.224300	-0.166400	-0.103700	-0.045830	0.0	-0.045830	-0.103700	-0.166400	-0.224300
54	-0.219500	-0.173700	-0.111000	-0.053070	0.0	-0.053070	-0.111000	-0.173700	-0.219500
59	-0.221900	-0.176100	-0.118200	-0.057890	0.0	-0.057890	-0.118200	-0.176100	-0.221900
64	-0.231600	-0.178500	-0.123000	-0.062710	0.0	-0.062710	-0.123000	-0.178500	-0.231600
69	-0.236400	-0.178500	-0.120600	-0.067540	0.0	-0.067540	-0.120600	-0.178500	-0.236400
74	-0.236400	-0.180900	-0.120600	-0.065130	0.0	-0.065130	-0.120600	-0.180900	-0.236400
79	-0.238800	-0.188100	-0.125400	-0.065130	0.0	-0.065130	-0.125400	-0.188100	-0.238800
84	-0.241200	-0.195400	-0.132700	-0.074770	0.0	-0.074770	-0.132700	-0.195400	-0.241200
89	-0.246000	-0.197800	-0.142300	-0.065130	0.0	-0.065130	-0.142300	-0.197800	-0.246000

F-4

CYDR(ALPHA,BETA)

BETA: ALPHA	-40	-30	-20	-10	0	10	20	30	40
-1	0.001670	0.001670	0.001670	0.001670	0.001670	0.001670	0.001670	0.001670	0.001670
4	0.001690	0.001690	0.001690	0.001690	0.001690	0.001690	0.001690	0.001690	0.001690
9	0.001660	0.001660	0.001660	0.001660	0.001660	0.001660	0.001660	0.001660	0.001660
14	0.001600	0.001600	0.001600	0.001600	0.001600	0.001600	0.001600	0.001600	0.001600
19	0.001490	0.001490	0.001490	0.001490	0.001490	0.001490	0.001490	0.001490	0.001490
24	0.000960	0.000960	0.001220	0.001220	0.001390	0.001220	0.001220	0.000960	0.000960
29	0.000590	0.000590	0.000880	0.000880	0.001240	0.000880	0.000880	0.000590	0.000590
34	0.000500	0.000500	0.000570	0.000570	0.001010	0.000570	0.000570	0.000500	0.000500
39	0.000410	0.000410	0.000230	0.000230	0.000440	0.000230	0.000230	0.000410	0.000410
44	0.000200	0.000200	0.000060	0.000006	0.000060	0.000006	0.000060	0.000200	0.000200
49	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
54	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
59	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
64	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
69	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
74	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
79	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
84	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
89	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

F-4

CX (ALPHA,BETA)

BETA: ALPHA	-40	-30	-20	-10	0	10	20	30	40
-1	-0.008500	-0.014000	-0.024500	-0.029000	-0.035500	-0.029000	-0.024500	-0.014000	-0.008500
4	-0.003500	-0.010000	-0.015000	-0.015000	-0.020000	-0.015000	-0.015000	-0.010000	-0.003500
9	0.003500	-0.003500	-0.002500	0.002500	0.002000	0.002500	-0.002500	-0.003500	0.003500
14	0.008500	0.003000	0.006500	0.009000	0.009000	0.009000	0.006500	0.003000	0.008500
19	0.015500	0.009500	0.009000	-0.006500	-0.015000	-0.006500	0.009000	0.009500	0.015500
24	0.021000	0.013500	-0.000500	-0.026500	-0.033500	-0.026500	-0.000500	0.013500	0.021000
29	0.019000	0.013000	-0.016000	-0.030500	-0.031000	-0.030500	-0.016000	0.013000	0.019000
34	0.018000	0.006000	-0.024000	-0.029000	-0.030500	-0.029000	-0.024000	0.006000	0.018000
39	0.022000	-0.001000	-0.021000	-0.024000	-0.032500	-0.024000	-0.021000	-0.001000	0.022000
44	0.021500	-0.003000	-0.012000	-0.015500	-0.021500	-0.015500	-0.012000	-0.003000	0.021500
49	0.008500	0.000500	0.004500	-0.002500	-0.007500	-0.002500	0.004500	0.000500	0.008500
54	0.009500	0.007500	0.013500	0.010000	0.011500	0.010000	0.013500	0.007500	0.009500
59	0.015000	0.016000	0.018500	0.020500	0.030000	0.020500	0.018500	0.016000	0.015000
64	0.021500	0.023500	0.024000	0.029000	0.043000	0.029000	0.024000	0.023500	0.021500
69	0.030500	0.027500	0.025500	0.026000	0.048000	0.026000	0.025500	0.027500	0.030500
74	0.039500	0.025500	0.024000	0.033000	0.047000	0.033000	0.024000	0.025500	0.039500
79	0.045500	0.033500	0.026000	0.030500	0.040000	0.030500	0.026000	0.033500	0.045500
84	0.039500	0.027500	0.025500	0.027000	0.025000	0.027000	0.025500	0.027500	0.039500
89	0.028500	0.039000	0.034000	0.039000	-0.005000	0.039000	0.034000	0.039000	0.028500

F-4

CZDS(ALPHA,BETA)

BETA:	-40	-30	-20	-10	0	10	20	30	40
ALPHA									
-1	-0.007000	-0.007550	-0.008100	-0.008000	-0.007900	-0.008000	-0.008100	-0.007550	-0.007000
4	-0.006700	-0.006000	-0.008100	-0.007850	-0.007600	-0.007850	-0.008100	-0.006000	-0.006700
9	-0.006300	-0.007100	-0.007900	-0.007600	-0.007300	-0.007600	-0.007900	-0.007100	-0.006300
14	-0.005700	-0.006500	-0.007300	-0.007200	-0.007100	-0.007200	-0.007300	-0.006500	-0.005700
19	-0.004800	-0.005800	-0.006700	-0.006900	-0.007100	-0.006900	-0.006700	-0.005800	-0.004800
24	-0.003900	-0.004950	-0.006000	-0.006700	-0.007400	-0.006700	-0.006000	-0.004950	-0.003900
29	-0.003600	-0.004500	-0.005400	-0.006600	-0.007700	-0.006600	-0.005400	-0.004500	-0.003600
34	-0.003800	-0.004350	-0.004900	-0.005900	-0.006800	-0.005900	-0.004900	-0.004350	-0.003800
39	-0.003300	-0.003900	-0.004500	-0.004650	-0.004800	-0.004650	-0.004500	-0.003900	-0.003300
44	-0.002600	-0.003250	-0.003900	-0.004000	-0.004100	-0.004000	-0.003900	-0.003250	-0.002600
49	-0.001800	-0.002600	-0.003400	-0.003570	-0.003750	-0.003570	-0.003400	-0.002600	-0.001800
54	-0.001200	-0.001950	-0.002700	-0.003000	-0.003300	-0.003000	-0.002700	-0.001950	-0.001200
59	-0.001450	-0.001850	-0.002250	-0.002370	-0.002500	-0.002370	-0.002250	-0.001850	-0.001450
64	-0.002600	-0.002350	-0.002100	-0.001950	-0.001800	-0.001950	-0.002100	-0.002350	-0.002600
69	-0.002350	-0.002620	-0.002900	-0.002150	-0.001400	-0.002150	-0.002900	-0.002620	-0.002350
74	-0.001200	-0.001950	-0.002700	-0.002050	-0.001400	-0.002050	-0.002700	-0.001950	-0.001200
79	-0.001000	-0.001950	-0.002900	-0.002250	-0.001600	-0.002250	-0.002900	-0.001950	-0.001000
84	-0.000600	-0.001700	-0.002800	-0.002400	-0.002000	-0.002400	-0.002800	-0.001700	-0.000600
89	-0.001700	-0.002100	0.002550	-0.002520	-0.002400	-0.002520	0.002550	-0.002100	-0.001700

F-4

CXDS(ALPHA,BETA)

BETA:	-40	-30	-20	-10	0	10	20	30	40
ALPHA									
-1	0.001530	0.001530	0.001530	0.001600	0.001600	0.001600	0.001530	0.001530	0.001530
4	-0.001520	0.001520	0.001520	0.001600	0.001600	0.001600	0.001520	0.001520	-0.001520
9	0.001300	0.001300	0.001300	0.001450	0.001450	0.001450	0.001300	0.001300	0.001300
14	-0.000990	0.000990	0.000990	0.001150	0.001150	0.001150	0.000990	0.000990	-0.000990
19	0.000500	0.000500	0.000500	0.000750	0.000750	0.000750	0.000500	0.000500	0.000500
24	0.000100	0.000100	0.000100	0.000300	0.000300	0.000300	0.000100	0.000100	0.000100
29	-0.000250	-0.000250	-0.000250	-0.000400	-0.000250	-0.000400	-0.000250	-0.000250	-0.000250
34	-0.000500	-0.001100	-0.001100	-0.000800	-0.000800	-0.000800	-0.001100	-0.001100	-0.000500
39	-0.000800	-0.001250	-0.001250	-0.001200	-0.000140	-0.001200	-0.001250	-0.001250	-0.000800
44	-0.001050	-0.001400	-0.001400	-0.001400	-0.002000	-0.001400	-0.001400	-0.001400	-0.001050
49	-0.001350	-0.001600	-0.001600	-0.001800	-0.002200	-0.001800	-0.001600	-0.001600	-0.001350
54	-0.001600	-0.001800	-0.001800	-0.002300	-0.002150	-0.002300	-0.001800	-0.001800	-0.001600
59	-0.001770	-0.002000	-0.002000	-0.002300	-0.001900	-0.002300	-0.002000	-0.002000	-0.001770
64	-0.002000	-0.002250	-0.002250	-0.002150	-0.001800	-0.002150	-0.002250	-0.002250	-0.002000
69	-0.002100	-0.002450	-0.002450	-0.002350	-0.002370	-0.002350	-0.002450	-0.002450	-0.002100
74	-0.002000	-0.002600	-0.002600	-0.002300	-0.002150	-0.002300	-0.002600	-0.002600	-0.002000
79	-0.002000	-0.002750	-0.002750	-0.002750	-0.002500	-0.002750	-0.002750	-0.002750	-0.002000
84	-0.002100	-0.002860	-0.002860	-0.003000	-0.003400	-0.003000	-0.002860	-0.002860	-0.002100
89	-0.002300	-0.002960	-0.002960	-0.003050	-0.004800	-0.003050	-0.002960	-0.002960	-0.002300

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CLDR(ALPHA,BETA)

ALPHA \ BETA:	-40	-30	-20	-10	0	10	20	30	40
-1	0.000035	0.000035	0.000090	0.000136	0.000184	0.000136	0.000090	0.000035	0.000035
4	0.000067	0.000067	0.000170	0.000155	0.000183	0.000155	0.000170	0.000067	0.000067
9	0.001000	0.000100	0.000232	0.000166	0.000176	0.000166	0.000232	0.000100	0.001000
14	0.000087	0.000087	0.000224	0.000170	0.000157	0.000170	0.000224	0.000087	0.000087
19	0.000010	0.000010	0.000010	0.000166	0.000800	0.000166	0.000010	0.000010	0.000010
24	0.0	0.0	0.0	0.000120	0.000033	0.000120	0.0	0.0	0.0
29	0.0	0.0	0.0	0.000010	0.000003	0.000010	0.0	0.0	0.0
34	0.0	0.0	0.0	0.000010	0.0	0.000010	0.0	0.0	0.0
39	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
44	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
49	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
54	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
59	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
64	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
69	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
74	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
79	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
84	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
89	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

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CLDA(ALPHA,BETA)

ALPHA \ BETA:	-40	-30	-20	-10	0	10	20	30	40
-1	-0.000720	-0.000750	-0.000680	-0.000770	-0.000760	-0.000770	-0.000680	-0.000750	-0.000720
4	-0.000780	-0.000790	-0.000800	-0.000830	-0.000800	-0.000830	-0.000800	-0.000790	-0.000780
9	-0.000670	-0.000720	-0.000640	-0.000740	-0.000730	-0.000740	-0.000640	-0.000720	-0.000670
14	-0.000550	-0.000620	-0.000460	-0.000530	-0.000520	-0.000530	-0.000460	-0.000620	-0.000550
19	-0.000380	-0.000500	-0.000350	-0.000340	-0.000360	-0.000340	-0.000350	-0.000500	-0.000380
24	-0.000215	-0.000350	-0.000160	-0.000210	-0.000230	-0.000210	-0.000150	-0.000350	-0.000215
29	-0.000100	-0.000200	-0.000130	-0.000130	-0.000150	-0.000130	-0.000130	-0.000200	-0.000100
34	-0.000050	-0.000110	-0.000050	-0.000090	-0.000099	-0.000090	-0.000050	-0.000110	-0.000050
39	-0.000030	-0.000060	-0.000050	-0.000060	-0.000060	-0.000060	-0.000050	-0.000060	-0.000030
44	0.000001	-0.000030	-0.000002	-0.000040	-0.000030	-0.000040	-0.000002	-0.000030	0.000001
49	0.000025	-0.000010	-0.000020	-0.000030	-0.000010	-0.000030	-0.000020	-0.000010	0.000025
54	0.000040	-0.000010	0.0	-0.000020	0.0	-0.000020	0.0	-0.000010	0.000040
59	0.000050	0.0	0.000010	-0.000010	0.000010	-0.000010	0.000010	0.0	0.000050
64	0.000070	0.0	0.000010	0.0	0.000020	0.0	0.000010	0.0	0.000070
69	0.000070	0.000010	0.000020	0.000015	0.000020	0.000015	0.000020	0.000010	0.000070
74	0.000070	0.000010	0.000020	0.000030	0.000020	0.000030	0.000020	0.000010	0.000070
79	0.000070	0.000020	0.000030	0.000040	0.000020	0.000040	0.000030	0.000020	0.000070
84	0.000070	0.000010	0.000020	0.000040	0.000020	0.000040	0.000020	0.000010	0.000070
89	0.000070	0.000020	0.000030	0.000050	0.000020	0.000050	0.000030	0.000020	0.000070

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CFY1(ALPHA,BETA)

BETA: ALPHA	-40	-30	-20	-10	0	10	20	30	40
-10	0.547000	0.436000	0.324000	0.145000	0.0	-0.145000	-0.324000	-0.436000	-0.547000
-5	0.585000	0.464000	0.343000	0.158000	0.0	-0.158000	-0.343000	-0.464000	-0.585000
0	0.583000	0.467000	0.351000	0.167000	0.0	-0.167000	-0.351000	-0.467000	-0.583000
5	0.572000	0.459000	0.346000	0.173000	0.0	-0.173000	-0.346000	-0.459000	-0.572000
10	0.543000	0.443000	0.342000	0.180000	0.0	-0.180000	-0.342000	-0.443000	-0.543000
15	0.506000	0.430000	0.353000	0.186000	0.0	-0.186000	-0.353000	-0.430000	-0.506000
20	0.471000	0.402000	0.332000	0.189000	0.0	-0.189000	-0.332000	-0.402000	-0.471000
25	0.484000	0.390000	0.295000	0.161000	0.0	-0.161000	-0.295000	-0.390000	-0.484000
30	0.544000	0.420000	0.295000	0.139000	0.0	-0.139000	-0.295000	-0.420000	-0.544000
35	0.618000	0.484000	0.350000	0.172000	0.0	-0.172000	-0.350000	-0.484000	-0.618000
40	0.672000	0.541000	0.410000	0.219000	0.0	-0.219000	-0.410000	-0.541000	-0.672000
45	0.652000	0.532000	0.412000	0.254000	0.0	-0.254000	-0.412000	-0.532000	-0.652000
50	0.612000	0.503000	0.394000	0.239000	0.0	-0.239000	-0.394000	-0.503000	-0.612000
55	0.618000	0.496000	0.373000	0.190000	0.0	-0.190000	-0.373000	-0.496000	-0.618000
60	0.670000	0.517000	0.363000	0.153000	0.0	-0.153000	-0.363000	-0.517000	-0.670000
65	0.715000	0.574000	0.433000	0.120000	0.0	-0.120000	-0.433000	-0.574000	-0.715000
70	0.681000	0.588000	0.494000	0.174000	0.0	-0.174000	-0.494000	-0.588000	-0.681000
75	0.638000	0.565000	0.492000	0.247000	0.0	-0.247000	-0.492000	-0.565000	-0.638000
80	0.619000	0.551000	0.482000	0.289000	0.0	-0.289000	-0.482000	-0.551000	-0.619000
85	0.615000	0.546000	0.476000	0.310000	0.0	-0.310000	-0.476000	-0.546000	-0.615000
90	0.620000	0.546000	0.471000	0.317000	0.0	-0.317000	-0.471000	-0.546000	-0.620000

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CMX1(ALPHA,BETA)

BETA: ALPHA	-40	-30	-20	-10	0	10	20	30	40
-10	-0.208800	-0.118500	-0.027700	0.006800	0.0	-0.006800	0.027700	0.118500	0.208800
-5	-0.076700	-0.020900	0.041800	0.020900	0.0	-0.020900	-0.041800	0.020900	0.076700
0	0.167100	0.160300	0.153500	0.055900	0.0	-0.055900	-0.153500	-0.160300	-0.167100
5	0.355500	0.320500	0.278800	0.118500	0.0	-0.118500	-0.278800	-0.320500	-0.355500
10	0.529900	0.418200	0.306500	0.139400	0.0	-0.139400	-0.306500	-0.418200	-0.529900
15	0.669300	0.474000	0.278800	0.139400	0.0	-0.139400	-0.278800	-0.474000	-0.669300
20	0.669300	0.453100	0.230200	0.132600	0.0	-0.132600	-0.230200	-0.453100	-0.669300
25	0.522600	0.327300	0.132600	0.111700	0.0	-0.111700	-0.132600	-0.327300	-0.522600
30	0.453100	0.272000	0.083500	0.048600	0.0	-0.048600	-0.083500	-0.272000	-0.453100
35	0.355500	0.230200	0.104400	-0.041800	0.0	0.041800	-0.104400	-0.230200	-0.355500
40	0.264700	0.160300	0.055900	-0.048600	0.0	0.048600	-0.055900	-0.160300	-0.264700
45	0.418200	0.257900	0.097600	-0.035000	0.0	0.035000	-0.097600	-0.257900	-0.418200
50	0.634300	0.453100	0.264700	0.014100	0.0	-0.014100	-0.264700	-0.453100	-0.634300
55	0.620200	0.487600	0.348200	0.125300	0.0	-0.125300	-0.348200	-0.487600	-0.620200
60	0.606100	0.487600	0.362300	0.195200	0.0	-0.195200	-0.362300	-0.487600	-0.606100
65	0.634300	0.501700	0.362300	0.202000	0.0	-0.202000	-0.362300	-0.501700	-0.634300
70	0.641100	0.501700	0.362300	0.188000	0.0	-0.188000	-0.362300	-0.501700	-0.641100
75	0.647900	0.508500	0.362300	0.181200	0.0	-0.181200	-0.362300	-0.508500	-0.647900
80	0.662000	0.501700	0.341400	0.188000	0.0	-0.188000	-0.341400	-0.501700	-0.662000
85	0.662000	0.501700	0.341400	0.202000	0.0	-0.202000	-0.341400	-0.501700	-0.662000
90	0.669300	0.508500	0.348200	0.222900	0.0	-0.222900	-0.348200	-0.508500	-0.669300

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CMY1(ALPHA,BETA)

BETA: ALPHA	-40	-30	-20	-10	0	10	20	30	40
-10	0.119000	0.155000	0.191000	0.269000	0.329000	0.269000	0.191000	0.155000	0.119000
-5	0.336000	0.246000	0.155000	0.189000	0.173000	0.189000	0.155000	0.246000	0.336000
0	0.420000	0.266000	0.113000	0.081000	0.063000	0.081000	0.113000	0.266000	0.420000
5	0.418000	0.225000	0.032000	-0.031000	-0.037000	-0.031000	0.032000	0.225000	0.418000
10	0.418000	0.160000	-0.098000	-0.142000	-0.148000	-0.142000	-0.098000	0.160000	0.418000
15	0.378000	0.074000	-0.229000	-0.222000	-0.218000	-0.222000	-0.229000	0.074000	0.378000
20	0.326000	0.012000	-0.299000	-0.307000	-0.284000	-0.307000	-0.299000	0.013000	0.326000
25	0.386000	0.030000	-0.326000	-0.359000	-0.401000	-0.359000	-0.326000	0.030000	0.386000
30	0.491000	0.083000	-0.324000	-0.343000	-0.531000	-0.343000	-0.324000	0.063000	0.491000
35	0.535000	0.124000	-0.287000	-0.377000	-0.579000	-0.377000	-0.287000	0.124000	0.535000
40	0.486000	0.121000	-0.244000	-0.493000	-0.603000	-0.493000	-0.244000	0.121000	0.486000
45	0.247000	0.037000	-0.174000	-0.115000	-0.617000	-0.115000	-0.174000	0.037000	0.247000
50	0.073000	-0.049000	-0.170000	-0.444000	-0.626000	-0.444000	-0.170000	-0.049000	0.073000
55	0.024000	-0.113000	-0.249000	-0.515000	-0.647000	-0.515000	-0.249000	-0.113000	0.024000
60	-0.185000	-0.299000	-0.412000	-0.638000	-0.703000	-0.638000	-0.412000	-0.299000	-0.185000
65	-0.494000	-0.558000	-0.621000	-0.705000	-0.805000	-0.705000	-0.621000	-0.558000	-0.494000
70	-0.719000	-0.773000	-0.826000	-0.868000	-0.953000	-0.868000	-0.826000	-0.773000	-0.719000
75	-0.868000	-0.975000	-1.082000	-1.082000	-1.136000	-1.082000	-1.082000	-0.975000	-0.868000
80	-1.000000	-1.186000	-1.371000	-1.294000	-1.328000	-1.294000	-1.371000	-1.186000	-1.000000
85	-1.135000	-1.351000	-1.567000	-1.541000	-1.619000	-1.541000	-1.567000	-1.351000	-1.135000
90	-1.274000	-1.487000	-1.700000	-1.811000	-1.974000	-1.811000	-1.700000	-1.487000	-1.274000

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CMZ1(ALPHA,BETA)

BETA: ALPHA	-40	-30	-20	-10	0	10	20	30	40
-10	-0.292900	-0.292900	-0.292900	-0.139400	0.0	0.139400	0.292900	0.292900	0.292900
-5	-0.292900	-0.299700	-0.306500	-0.146200	0.0	0.146200	0.306500	0.299700	0.292900
0	-0.272000	-0.285600	-0.292900	-0.153500	0.0	0.153500	0.292900	0.285600	0.272000
5	-0.257900	-0.264700	-0.272000	-0.153500	0.0	0.153500	0.272000	0.264700	0.257900
10	-0.243800	-0.237000	-0.237000	-0.146200	0.0	0.146200	0.237000	0.237000	0.243800
15	-0.202000	-0.257900	-0.167100	-0.111700	0.0	0.111700	0.167100	0.257900	0.202000
20	-0.035000	-0.020900	-0.006800	-0.055900	0.0	0.055900	0.006800	0.020900	0.035000
25	0.216100	0.195200	0.167100	0.048600	0.0	-0.048600	-0.167100	-0.195200	-0.216100
30	0.202000	0.222900	0.243800	0.167100	0.0	-0.167100	-0.243800	-0.222900	-0.202000
35	0.111700	0.174400	0.237000	0.195200	0.0	-0.195200	-0.237000	-0.174400	-0.111700
40	0.243800	0.222900	0.208800	0.174400	0.0	-0.174400	-0.208800	-0.222900	-0.243800
45	0.320500	0.230200	0.139400	0.118500	0.0	-0.118500	-0.139400	-0.230200	-0.320500
50	0.264700	0.111700	-0.035000	-0.104400	0.0	0.104400	0.035000	-0.111700	-0.264700
55	0.237000	0.048600	-0.139400	-0.272000	0.0	0.272000	0.139400	-0.048600	-0.237000
60	0.306500	0.083500	-0.139400	-0.334600	0.0	0.334600	0.139400	-0.083500	-0.306500
65	0.494900	0.292900	0.090800	-0.341400	0.0	0.341400	-0.090800	-0.292900	-0.494900
70	0.501700	0.369600	0.230200	-0.160300	0.0	0.160300	-0.230200	-0.369600	-0.501700
75	0.376400	0.264700	0.153500	-0.055900	0.0	0.055900	-0.153500	-0.264700	-0.376400
80	0.272000	0.153500	0.035000	-0.083500	0.0	0.083500	-0.035000	-0.153500	-0.272000
85	0.257900	0.111700	-0.027700	-0.097600	0.0	0.097600	0.027700	-0.111700	-0.257900
90	0.188000	0.062700	-0.062700	-0.104400	0.0	0.104400	0.062700	-0.062700	-0.138000

F111 F(ALPHA ONLY)

ALPHA	CFX1	CFZ1	CMX4	CMZ4	CMX2	CMZ2	CMX5	CMX6	CMZ5
-10	-0.009000	0.850000	0.278762	-1.184728	-0.975660	-0.069694	-0.005600	0.001400	-0.001400
-5	-0.025000	0.460000	0.278762	-1.184728	-1.324116	-0.069694	-0.005600	0.000100	-0.001400
0	-0.030000	0.080000	0.278762	-1.184728	-1.324116	-0.069694	-0.005600	0.001400	-0.001400
5	-0.028600	-0.320000	0.627218	-1.184728	-1.115047	-0.069694	-0.005600	0.001400	-0.000700
10	0.009800	-0.730000	0.905980	-1.254421	-1.254421	-0.069694	-0.006300	0.001400	-0.000700
15	0.045100	-1.130000	1.533183	-1.533183	-1.254421	0.0	-0.005600	0.000700	0.0
20	0.073100	-1.530000	2.160401	-1.811945	-1.115047	0.069694	-0.005600	0.000700	0.000700
25	0.099100	-1.920000	3.345127	-1.881639	-1.254421	1.324116	-0.006300	0.000700	0.000700
30	0.092000	-2.330000	4.460177	-1.951319	-1.811945	2.508842	-0.006300	0.001400	0.001400
35	0.070800	-2.650000	8.084065	-1.951319	-2.648232	4.042027	-0.006300	0.001400	0.002100
40	0.046500	-1.747000	11.080740	-1.254421	-3.832958	2.787604	-0.006300	0.002100	0.002100
45	0.039700	-1.689600	6.899324	-0.557524	-4.181412	1.811945	-0.006300	0.002800	0.002100
50	0.035400	-1.705400	2.439163	1.115047	-3.972347	1.324116	-0.004900	0.002800	0.002100
55	0.039200	-1.749200	1.742251	6.899324	-3.136060	0.975660	-0.004200	0.002100	0.002100
60	0.039700	-1.768000	0.905980	5.714595	-1.881639	-2.160401	-0.003500	0.000700	0.002100
65	0.034400	-1.814200	0.209068	0.0	-1.045354	-3.275435	-0.003500	0.000700	0.002100
70	0.035800	-1.902000	0.0	-0.766592	-0.696898	-0.348456	-0.002800	0.0	0.002100
75	0.039500	-1.949000	0.069694	0.766592	-0.696898	-1.045354	-0.003500	0.0	0.002100
80	0.040700	-1.963400	0.139374	0.766592	-0.926887	0.278762	-0.004200	0.0	0.002100
85	0.041200	-1.969000	0.069694	0.766592	-0.975660	-0.278762	-0.003500	-0.000700	0.002100
90	0.041200	-1.969000	0.069694	0.766592	-1.045354	-0.348456	-0.001400	-0.001400	0.002100

F111 F(ALPHA ONLY)

ALPHA	CFX7	CFZ7	CMY7	CMY3	CFY6	CFY5	CFY2	CFY4	CMZ6
-10	0.006100	-0.012700	-0.026400	-26.040000	0.003500	0.000500	0.030000	0.120000	-0.009800
-5	0.005500	-0.015600	-0.029700	-26.040000	0.003400	0.000500	0.060000	0.130000	-0.009100
0	0.005000	-0.010000	-0.030300	-26.040000	0.003200	0.011000	0.120000	0.160000	-0.009100
5	0.004300	-0.014800	-0.030300	-24.420000	0.003100	0.000600	0.190000	0.130000	-0.008400
10	0.003500	-0.014300	-0.030500	-22.790000	0.002900	0.000600	0.230000	0.010000	-0.009100
15	0.002600	-0.015400	-0.031200	-26.230000	0.003000	0.001000	0.240000	0.0	-0.009100
20	0.001700	-0.018400	-0.032000	-29.670000	0.003200	0.001300	0.230000	0.430000	-0.009100
25	0.000800	-0.020700	-0.034400	-33.700000	0.003300	0.001800	0.260000	1.050000	-0.009100
30	-0.000300	-0.024100	-0.037000	-37.720000	0.003200	0.001400	0.290000	1.200000	-0.009100
35	-0.001200	-0.025000	-0.036100	-42.900000	0.002900	0.001300	0.290000	0.790000	-0.008400
40	-0.002100	-0.022000	-0.031600	-44.690000	0.002500	0.002700	0.560000	0.230000	-0.006300
45	-0.003500	-0.016900	-0.024700	-41.810000	0.001900	0.001400	1.230000	-0.180000	-0.004900
50	-0.004300	-0.014400	-0.017400	-19.000000	0.001900	0.000700	1.700000	0.780000	-0.004200
55	-0.004800	-0.012000	-0.011400	2.000000	0.004100	0.000200	1.540000	2.610000	-0.004200
60	-0.004500	-0.010300	-0.008700	-7.000000	0.001600	-0.001300	-0.140000	2.270000	-0.004200
65	-0.004700	-0.009300	-0.003400	-30.000000	-0.000500	-0.003500	-1.180000	0.490000	-0.004200
70	-0.005200	-0.010500	-0.004000	-27.000000	0.000400	-0.003500	-0.090000	-0.180000	-0.004200
75	-0.005300	-0.010600	-0.004000	-4.000000	0.000500	-0.002700	0.640000	-0.010000	-0.004200
80	-0.005200	-0.008200	-0.004000	-3.000000	0.0	-0.002500	0.580000	0.090000	-0.004200
85	-0.005600	-0.007100	-0.004000	-17.000000	-0.000500	-0.002400	0.610000	0.100000	-0.004200
90	-0.061000	-0.007700	-0.004000	-24.000000	-0.000800	-0.002400	0.730000	0.160000	-0.004200

F-4 F(ALPHA ONLY)

ALPHA	CYP	CLR	CLP	CNR	CNP	CMQ	CYR
-1	0.193000	0.024120	-0.506500	-1.061000	0.0	-4.700000	2.050000
4	0.699500	0.108500	-0.651300	-0.844200	-0.084420	-4.800000	1.881000
9	0.482400	0.241200	-0.639200	-0.916600	-0.048240	-5.200000	1.761000
14	0.554800	0.482400	-0.542700	-0.771900	-0.024120	-6.200000	1.423000
19	0.916600	1.085000	-0.603000	-1.206000	0.048240	-6.100000	1.544000
24	1.471000	1.713000	-0.964800	-1.857000	0.072360	-7.100000	1.110000
29	1.930000	1.375000	-0.964800	-1.206000	0.048240	-6.900000	-0.964800
34	1.254000	0.964800	-0.880400	-0.458300	-0.120600	-9.200000	-2.653000
39	-0.048240	0.603000	-0.482400	-0.193000	-0.313600	-10.000000	-2.412000
44	-1.134000	0.325600	-0.337700	-0.217100	-0.337700	-11.200000	-1.978000
49	-0.651300	0.120600	-0.289400	-0.482400	-0.144700	-10.500000	-1.930000
54	-1.688000	0.036180	-0.241200	-0.940700	-0.385900	-8.800000	-0.964800
59	-2.002000	0.048240	-0.313600	-0.964800	-0.554800	-7.200000	-2.388000
64	-1.182000	0.096480	-0.410000	-0.868300	-0.361800	-7.100000	-2.002000
69	-1.688000	0.193000	-0.518600	-0.578900	-0.120600	-6.700000	-1.254000
74	-1.110000	0.193000	-0.639200	-0.120600	0.072360	-7.100000	-0.699500
79	-1.013000	0.108500	-0.735700	0.434200	0.337700	-6.600000	-0.385900
84	-1.664000	0.024120	-0.844200	0.603000	0.554800	-6.500000	-0.506500
89	-1.930000	0.0	-0.916600	0.603000	0.892500	-6.200000	-0.892500

THE BELOW LISTED COEFFICIENTS HAVE BEEN EQUATED TO ZERO

CYDA	CXR	CXDR	CXP	CXQ	CXDA	CYQ
CYDH	CZP	CZQ	CZE	CZDA	CZDR	CLQ
CLDS	CMP	CMR	CMDA	CMDR	CNQ	CNDH

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Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

INATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

ORT TITLE

The Numerical Solution and Analysis of Airplane Spin Equations Modeled in a Fixed Coordinate System

CRIPTIVE NOTES (Type of report and inclusive dates)

Master's Thesis; December 1972

HOR(S) (First name, middle initial, last name)

Robert Louis Champoux

ORT DATE December 1972	7a. TOTAL NO. OF PAGES 140	7b. NO. OF REFS 8
NTRACT OR GRANT NO. ROJECT NO.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	

STRIBUTION STATEMENT

Approved for public release; distribution unlimited

UPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California 93940

BSTRACT

Three forms of the airplane spin equations of motion, derived by Buehler in Reference [1], form the basis for the development of a computer program designed to seek dynamically stable equilibrium solutions of a spinning aircraft. The program incorporates two solution techniques: one based upon Euler integration, the other, a version of minimization by gradient search. Secondary programs are developed to (1) generate power off glide parameters for use in the validation of the equations of motion, and (2) evaluate equation residuals obtained from a grid of initial conditions over the potential solution space. F-111 and F-4 aerodynamic force and moment models were utilized to evaluate the solution methods and equations of motion. The numerical results indicate that the F-111 and F-4 data are not representative of the actual aircraft and, therefore, it is highly unlikely that dynamically stable equilibrium solutions can be achieved from these models. The utility of the two solution methods is evaluated and the numerical results are analyzed in order to gain insight into the optimal application of the three forms of the equations of motion. The paper concludes with a discussion concerning the qualitative validation of the equations of motion.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Airplane Spin Equations						
F-4 Spin						
F-111 Spin						
Gradient Search Solution Method						

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